[08-10-31-T10]

Assignment on complex numbers

The following summarizes our discussion today. If you visit the homework page at math.mnrt.net you will find a link to photos of our chalkboards at the end of class today.

Please carefully consider the following four questions and write your answers to them on paper. Due at our next class. You do not have to think about and answer them in the order in which they are listed here.

■ [1] Consider the following "proof" that $-1 = 1$.

The following identity holds for all values of x and y:

$$\sqrt{x-y} = i\sqrt{y-x} \tag{1}$$

Setting $x = a$, $y = b$, $a \neq b$, we find

$$\sqrt{a-b} = i\sqrt{b-a} \tag{2}$$

Now setting $x = b$, $y = a$, we find

$$\sqrt{b-a} = i\sqrt{a-b} \tag{3}$$

Multiplying the last two equations, member by member, we get

$$\sqrt{a-b}\,\sqrt{b-a} = i^2\sqrt{b-a}\,\sqrt{a-b} \tag{4}$$

Dividing both sides by $\sqrt{a-b}\,\sqrt{b-a}$, we finally get

$1 = i^2$, or what is equivalent $1 = -1$.

□

[Question 1] Explain what is wrong with the following "proof" that $-1 = 1$.

■ [2] Mischa's proposed counterexample to EQ1

Consider $\sqrt{x-y} = i\sqrt{y-x}$ with $x = 5$, $y = 3$

$$\sqrt{5-3} = i\sqrt{3-5}$$
$$\sqrt{2} = i\sqrt{-2}$$
$$\sqrt{2} = i^2\sqrt{2}$$
$$\sqrt{2} = -\sqrt{2}$$

But $\sqrt{2} = -\sqrt{2}$ is absurd. So EQ1 is not true for all numbers x, y .

[Question 2] What, if anything, is wrong with this alleged counterexample?

■ [3] Christine's proof that EQ1 is false

Suppose that $\sqrt{x-y} = i\sqrt{y-x}$ is true for all x, y. Then

$$\sqrt{x-y} = i\sqrt{y-x}$$
$$\Longrightarrow \sqrt{x-y} = i\sqrt{(-1)(x-y)}$$
$$\Longrightarrow \sqrt{x-y} = i\sqrt{-1}\ \sqrt{(x-y)}$$
$$\Longrightarrow \sqrt{x-y} = i^2\sqrt{(x-y)}$$
$$\Longrightarrow \sqrt{x-y} = -\sqrt{(x-y)}$$

This is true only in the case that $x = 0$, $y = y$, but in all other cases clearly false. Therefore, the supposition that $\sqrt{x-y} = i\sqrt{y-x}$ for all x, y is false.

[Question 3] What, if anything, is wrong with this alleged proof?

■ [4] Addittional question

$$\sqrt{2} = \sqrt{2}$$
$$\Longrightarrow \sqrt{2} = i\sqrt{-2}$$
$$\Longrightarrow \sqrt{2} = i^2\sqrt{2}$$
$$\therefore \sqrt{2} = -\sqrt{2}$$

[Question 4] What went wrongf?