

Exercises <sup>[A-2]</sup>

In exercises 1–6 find (a) the nature of the roots, (b) the sum of the roots, (c) the product of the roots.

1.  $3x^2 - 10x + 9 = 0$

4.  $2t(1-t) = 1$

2.  $12x = 3 + 8x^2$

5.  $3x^2 + 1.2 = 4.5x$

3.  $2.25x^2 = 6x - 4$

6.  $2x^2 + 6kx + 3k^2 = 0$

7. Show that  $2x^2 - 8x + 8$  has equal zeros, and sketch the graph of the function defined by  $y = 2x^2 - 8x + 8$ .

8. Find the value of  $k$  for which  $4x^2 + 5x + k$  is a trinomial square.

9. Solve the equation  $3x^2 - 5x + 3 = 0$ . Use the sum and product of the roots as a check.

10. Solve the equation  $4x(x+2) = -1$ . Use the sum and product of the roots as a check.

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## CHAPTER 13

11. The equation  $\frac{x^2}{x-1} = \frac{b^2}{b-1}$  has an obvious solution  $x = b$ . Use the product of the roots of the equation to obtain the other root, and check by considering the sum of the roots.

12. Is  $x = 2$  a solution of the equation  $\frac{x^2}{4} = \frac{x+a}{2+a}$ ? Complete the solution and check as in exercise 11.

13. If  $a$  and  $c$  have opposite signs in the equation  $ax^2 + bx + c = 0$ , what may be deduced about the roots of the equation?

14. State the condition which ensures that  $ax^2 + bx + c$  shall have linear factors with rational coefficients.

15. State the sum of the roots of  $x^2 - 5x + k = 0$ . If the difference of the roots is 1, find the roots and the value of  $k$ .

16. State the sum of the roots of  $2x^2 + 7x + k = 0$ . If the roots differ by 1.5, find the roots and the value of  $k$ .

17. If the roots of  $3x^2 - 8x + k = 0$  are in the ratio 3 : 1, find  $k$ .

18. Show that the discriminant of  $(k+1)x^2 + 4kx + 2 = 0$  defines a quadratic function of  $k$ . Find the zeros of this function and make a sketch of its graph. For what values of  $k$  are the roots of the given equation (a) equal, (b) real and unequal?

19. Show that the roots of  $4x^2 + 4mx + 2m - 1 = 0$  are real for all values of  $m$ . Solve the equation using the value of  $m$  which makes the roots equal.

20. Show that the roots of  $k^2(x^2 - 1) + 2kx + 10 = 0$  are real except for values of  $k$  between  $-3$  and  $3$ .

21. Find the values of  $k$  for which the roots of  $(kx+1)^2 = (k+1)x$  are (a) equal, (b) real and unequal.

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<u>a</u>	<u>b</u>	<u>c</u>
<u>1.</u> Conjugate imaginary nos.	$3\frac{1}{3}$	3
<u>2.</u> Real, unequal, irrational	$1\frac{1}{2}$	$\frac{3}{8}$
<u>3.</u> Equal, rational	$\frac{8}{3}$	$\frac{16}{9}$
<u>4.</u> Conjugate imaginary nos.	1	$\frac{1}{2}$
<u>5.</u> Real, unequal, irrational	1.5	0.4
<u>6.</u> Real, unequal, irrational	-3k	$\frac{3}{2}k^2$
<u>7.</u> D = 0	<u>17.</u> k = 4	
<u>8.</u> k = $\frac{25}{16}$	<u>18.</u> D = $16k^2 - 8k - 8$ ;	
<u>9.</u> $\left\{ \frac{5}{6} + \frac{i\sqrt{11}}{6}, \frac{5}{6} - \frac{i\sqrt{11}}{6} \right\}$	$\left\{ -\frac{1}{2}, 1 \right\}$	
<u>10.</u> $\left\{ -1 + \frac{1}{2}\sqrt{3}, -1 - \frac{1}{2}\sqrt{3} \right\}$	<u>a.</u> $\left\{ -\frac{1}{2}, 1 \right\}$	
<u>11.</u> Other root, $\frac{b}{b-1}$ ;	<u>b.</u> $\{k k < -\frac{1}{2}\} \cup \{k k > 1\}$	
product = $\frac{b^2}{b-1}$ ; sum = $\frac{b^2}{b-1}$	<u>19.</u> D = $16(m-1)^2$ ;	
<u>12.</u> Yes; other root, $\frac{-2a}{2+a}$	$\left\{ -\frac{1}{2}, -\frac{1}{2} \right\}$	
<u>13.</u> They have opposite signs	<u>20.</u> D = $4k^2(k^2 - 9)$	
<u>14.</u> $b^2 - 4ac = k^2$ , k rational	<u>21.</u> <u>a.</u> $\left\{ \frac{1}{3}, -1 \right\}$	
<u>15.</u> 5; {3, 2}; k = 6	<u>b.</u> $\{k -1 < k < \frac{1}{3}, k \neq 0\}$	
<u>16.</u> -3.5; {-1, -2.5}; k = 5		