

Exercises [A-1]

Perform the indicated operations:

1. $3\sqrt{-4} + 2\sqrt{-9} - \sqrt{-64}$
2. $2\sqrt{-8} + 4\sqrt{-\frac{1}{2}} - \sqrt{-50}$
3. $(2 + \sqrt{-9}) + (3 - \sqrt{-4})$
4. $(4 + \sqrt{-1}) - (3 + \sqrt{-4})$
5. $(3 + 4i) + (3 - 4i)$
6. $(3 + 4i) - (3 - 4i)$
7. $(\frac{1}{2} + \sqrt{-\frac{1}{2}}) - (\frac{1}{8} + \sqrt{-\frac{1}{8}})$
8. $(1 - \sqrt{-\frac{1}{3}}) - \frac{1}{2}(1 - \sqrt{-3})$
9. $i + i^2 + i^3 + i^4$
10. $\frac{1}{2}(2 - i\sqrt{3}) + \frac{1}{2}(2 + i\sqrt{3})$
11. $\sqrt{-4} \cdot \sqrt{-9}$
12. $\sqrt{-4} \cdot \sqrt{-\frac{1}{2}}$
13. $\sqrt{-1\frac{1}{3}} \cdot \sqrt{-1\frac{1}{2}}$
14. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{3}{5}} \cdot \sqrt{-\frac{5}{8}}$
15. $6i \div 2i$
16. $i\sqrt{3} \div i\sqrt{2}$
17. $\sqrt{-18} \div \sqrt{-8}$
18. $\sqrt{-\frac{2}{3}} \div \sqrt{-2\frac{2}{3}}$
19. $(1 + i)(1 - i)$
20. $(1 - i)^2$
21. $(2 + i)(1 - i)$
22. $(3 - 2i)(3 + 2i)$
23. $(3 - 2i)^2$
24. $(2 + 3i)(5 + 3i)$
25. $(2 + i\sqrt{3})(2 - i\sqrt{3})$
26. $(-2 + i)(-1 - 2i)$
27. $(3 - i\sqrt{2})(2 + i\sqrt{2})$
28. $(2 + \sqrt{-2})(2 - \sqrt{-2})$
29. $(2 - \sqrt{-3})^2$
30. $\left(\frac{2 - i\sqrt{5}}{3}\right)\left(\frac{2 + i\sqrt{5}}{3}\right)$
31. $(1 - \frac{1}{2}i)(1 + \frac{1}{2}i)$
32. $\left(\frac{2 + i\sqrt{2}}{2}\right)\left(\frac{3 + i\sqrt{2}}{2}\right)$
33. $2 \div i$
34. $5 \div (3 + i)$
35. $3 \div (2 + \sqrt{-2})$
36. $(4 - 2i) \div (1 - i)$
37. $2i \div (2 + i\sqrt{2})$
38. $(3 + i) \div (3 - i)$
39. $(4 - 2i) \div (2 - i)$
40. $(3 + i\sqrt{3}) \div (1 - i\sqrt{3})$
41. Evaluate $x^2 - 4x$ when $x = 2 + i$.
42. Evaluate $2x^2 - 5x$ when $x = \frac{5}{4} - \frac{1}{4}i$.
43. If $f(x) = \frac{x}{3-x}$, evaluate $f(1 + 2i)$.
44. If $f(x) = \frac{x}{x^2 + 1}$, evaluate $f\left(\frac{-1 + i\sqrt{3}}{2}\right)$.
45. Show by substitution that $\frac{1}{2}(1 + i)$ is a root of $2x^2 - 2x + 1 = 0$.
46. Show by substitution that $\frac{3 - i\sqrt{3}}{3}$ is a root of $3x^2 - 6x + 4 = 0$.
47. (a) Solve $x^2 - 6x + 10 = 0$.
(b) Find the sum of the roots. (c) Find the product of the roots.

48. (a) Solve $3x^2 - 12x + 16 = 0$.
 (b) Find the sum of the roots. (c) Find the product of the roots.
49. (a) Solve $4x^2 + 6x + 5 = 0$.
 (b) Find the sum of the roots. (c) Find the product of the roots.
50. (a) Solve $9x^2 + 18x + 13 = 0$.
 (b) Find the sum of the roots. (c) Find the product of the roots.
51. If $x + iy = (2 + i\sqrt{2})^2$, find x and y .
52. If $x + iy = (4 - 7i) \div (2 - i)$, find x and y .
53. If $f(x) = x^2 - 6x + 5$, show that $f(3 + ik)$ is a real, negative number for every real value of k .
54. If $f(x) = 1 - 4x - x^2$, show that $f(-2 + ik)$ is a real, positive number for every real value of k .

Exercises ^[A-2]

Perform the indicated operations:

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|---|--|
| 1. $(2 - i)(3 - i)$ | 11. $3 \div 2i$ |
| 2. $(3 - \sqrt{-2})^2$ | 12. $-4 \div i\sqrt{2}$ |
| 3. $(\sqrt{3} + i\sqrt{2})(\sqrt{3} - i\sqrt{2})$ | 13. $(-3 + 2i) \div (2 - i)$ |
| 4. $(2 - i\sqrt{3})(3 - 2i\sqrt{3})$ | 14. $(3 - \sqrt{-3}) \div (1 - \sqrt{-3})$ |
| 5. $(1 + i)^2(1 - i)$ | 15. $(3 + 4i) \div (2 + i)$ |
| 6. $(2 + i\sqrt{2})(2 - i\sqrt{2})$ | 16. $5i \div (1 + 3i)$ |
| 7. $\left(\frac{1 - i\sqrt{3}}{2}\right)\left(\frac{1 + i\sqrt{3}}{2}\right)$ | 17. $(3 - 6i) \div (1 - 2i)$ |
| 8. $(1 - i)^3$ | 18. $(i + i^2) \div (i^3 + i^4)$ |
| 9. $(2 + i\sqrt{5})(3 - i\sqrt{5})$ | 19. $(\frac{1}{2} + \frac{1}{4}i) \div (\frac{3}{2} - \frac{1}{2}i)$ |
| 10. $\left(\frac{3 + i}{2}\right)\left(\frac{3 - i}{2}\right)$ | 20. $(\sqrt{2} + i)^2 \div (\frac{1}{2} + i\sqrt{2})$ |
21. Show that $2 - i$ is a square root of $3 - 4i$.
22. Show that $\frac{\sqrt{2}}{2}(1 + i)$ is a square root of i .
23. Evaluate $2x^2 + 6x$ when $x = -1.5 + 0.5i$.
24. Evaluate $3x^2 - 6x$ when $x = 1 - i\sqrt{3}$.
25. If $f(x) = \frac{2 + x}{2 - x}$, evaluate $f(1 - i)$.

26. If $f(x) = x + \frac{4}{x}$, evaluate $f(1 + i\sqrt{3})$.
27. (a) Evaluate $(1 + i)(\frac{1}{2} - \frac{1}{2}i)$.
 (b) State the reciprocal of $1 + i$ in the form $a + bi$.
 (c) Complete the statement $n \div (1 + i) = n \cdot (?)$.
28. (a) Evaluate $(2 - i)(\frac{2}{5} + \frac{1}{5}i)$.
 (b) State the reciprocal of $2 - i$ in the form $a + bi$.
 (c) Complete the statement $n \div (2 - i) = n \cdot (?)$.
29. (a) Show that $\frac{1}{2} - \frac{1}{2}i\sqrt{3}$ is the reciprocal of $\frac{1}{2} + \frac{1}{2}i\sqrt{3}$.
 (b) Complete the statement $n \div (\frac{1}{2} - \frac{1}{2}i\sqrt{3}) = n \cdot (?)$.
30. (a) Find the reciprocal of $a + ib$.
 (b) If $f(x) = x + \frac{a^2 + b^2}{x}$, show that $f(a + ib) = f(a - ib) = 2a$.
31. If $x + iy = (3 + i)^2$, find x and y , and show that $x - iy = (3 - i)^2$.
32. If $x + iy = (1 + i)^3$, find x and y , and show that $x - iy = (1 - i)^3$.
33. Solve $x^2 - 3x + 3 = 0$. Find the sum of the roots and the product of the roots.
34. Solve $2x^2 + 6x + 5 = 0$. Find the sum of the roots and the product of the roots.
35. Solve $5x^2 - 2x + 1 = 0$. Find the sum of the roots and the product of the roots.
36. Solve $3x^2 - 4x + 5 = 0$. Find the sum of the roots and the product of the roots.

The Nature of the Roots of $ax^2 + bx + c = 0$

In the complex number system, the quadratic equation $ax^2 + bx + c = 0$ has two roots, given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

for any values of a, b, c . The roots may be written

$$\frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}, \quad \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}.$$

If a, b, c are rational numbers, the term $\frac{-b}{2a}$ is a rational number. The term $\frac{\sqrt{b^2 - 4ac}}{2a}$, on the other hand, may be real or imaginary, rational or

Page 341

1. $4 - 2i$

2. $\frac{1}{2} + \frac{1}{2}i$

3. $2 + 2i$

4. $\frac{1}{2} + \frac{1}{2}i\sqrt{3}$

5. $\frac{3}{5} + \frac{1}{5}i$

6. $-\frac{\sqrt{2}}{2} + i$

7. $\frac{1}{2} + \frac{3}{2}i$

8. $-i\sqrt{2}$

9. $\frac{4}{25} - \frac{3}{25}i$

10. $\frac{1}{4} + \frac{\sqrt{3}}{12}i$

11. $\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Pages 343-344

1. $4i$

2. $i\sqrt{2}$

3. $5 + i$

4. $1 - i$

5. 6

6. $8i$

7. $\frac{3}{8} + \frac{1}{4}i\sqrt{2}$

8. $\frac{1}{2} + \frac{1}{6}i\sqrt{3}$

9. 0

10. 2

11. -6

12. $-\sqrt{2}$

13. $-\sqrt{2}$

14. $\frac{1}{2}i$

15. 3

16. $\frac{1}{2}\sqrt{6}$

17. $\frac{3}{2}$

18. $\frac{1}{2}$

19. 2

20. $-2i$

21. $3 - i$

22. 13

23. $5 - 12i$

24. $1 + 21i$

25. 7

26. $4 + 3i$

27. $8 + i\sqrt{2}$

28. 6

29. $1 - 4i\sqrt{3}$

30. 1

31. $\frac{5}{4}$

32. $1 + \frac{5}{4}i\sqrt{2}$

33. $-2i$

34. $\frac{3}{2} - \frac{1}{2}i$

35. $1 - \frac{1}{2}i\sqrt{2}$

36. $3 + i$

37. $\frac{\sqrt{2}}{3} + \frac{2}{3}i$

38. $\frac{4}{5} + \frac{3}{5}i$

39. 2

40. $i\sqrt{3}$

41. -5

42. $-\frac{13}{4}$

43. $-\frac{1}{4} + \frac{3}{4}i$

44. -1

47. a. $\{3 + i, 3 - i\}$

b. 6

c. 10

48. a. $\{2 + \frac{2}{3}i\sqrt{3}, 2 - \frac{2}{3}i\sqrt{3}\}$

b. 4

c. $\frac{16}{3}$

49. a. $\{-\frac{3}{4} + \frac{i\sqrt{11}}{4}, -\frac{3}{4} - \frac{i\sqrt{11}}{4}\}$

b. $-\frac{3}{2}$

c. $\frac{5}{4}$

50. a. $\{-1 + \frac{2}{3}i, -1 - \frac{2}{3}i\}$

b. -2

c. $\frac{13}{9}$

51. $x = 2; y = 4\sqrt{2}$

52. $x = 3; y = -2$

53. $-(k^2 + 4)$ is negative

54. $5 + k^2$ is positive

Pages 344-345

1. $5 - 5i$

2. $7 - 6i\sqrt{2}$

3. 5

4. $-7i\sqrt{3}$

5. $2 + 2i$

6. 6

7. 1

8. $-2 - 2i$

9. $11 + i\sqrt{5}$
10. $\frac{5}{2}$
11. $-\frac{3}{2}i$
12. $2i\sqrt{2}$
13. $-\frac{8}{5} + \frac{1}{5}i$
14. $\frac{3}{2} + \frac{1}{2}i\sqrt{3}$
15. $2 + i$
16. $\frac{3}{2} + \frac{1}{2}i$
17. 3
18. -1
19. $\frac{1}{4} + \frac{1}{4}i$
20. 2
23. -5
24. -12
25. $1 - 2i$
26. 2
27. a. 1
b. $\frac{1}{2} - \frac{1}{2}i$
c. $(\frac{1}{2} - \frac{1}{2}i)$
28. a. 1
b. $\frac{2}{5} + \frac{1}{5}i$
c. $(\frac{2}{5} + \frac{1}{5}i)$
29. b. $(\frac{1}{2} + \frac{1}{2}i\sqrt{3})$
30. a. $\frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}i$
31. $x = 8; y = 6$
32. $x = -2; y = 2$
33. $\{\frac{3}{2} + \frac{i\sqrt{3}}{2}, \frac{3}{2} - \frac{i\sqrt{3}}{2}\}$
Sum = 3
Product = 3
34. $\{-\frac{3}{2} + \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i\}$
Sum = -3
Product = $\frac{5}{2}$
35. $\{\frac{1}{5} + \frac{2}{5}i, \frac{1}{5} - \frac{2}{5}i\}$
Sum = $\frac{2}{5}$
Product = $\frac{1}{5}$
36. $\{\frac{2}{3} + \frac{i\sqrt{11}}{3}, \frac{2}{3} - \frac{i\sqrt{11}}{3}\}$
Sum = $\frac{4}{3}$
Product = $\frac{5}{3}$

Page 348

1. $D = -7$; conjugate imaginary numbers.
2. $D = 9$; real, unequal, rational.
3. $D = 76$; real, unequal, irrational.
4. $D = 0$; equal, rational.
5. $D = 17$; real, unequal, irrational.
6. $D = -32$; conjugate imaginary numbers.

