

## 7.2 TRIGONOMETRIC INTEGRALS

In this section we use trigonometric identities to integrate certain combinations of trigonometric functions. We start with powers of sine and cosine.

**EXAMPLE 1** Evaluate  $\int \cos^3 x \, dx$ .

**SOLUTION** Here the appropriate identity is  $\cos^2 x = 1 - \sin^2 x$ . We write

$$\cos^3 x = \cos^2 x \cdot \cos x = (1 - \sin^2 x) \cos x$$

It is useful to have the extra factor of  $\cos x$  because if we make the substitutor  $u = \sin x$ , then we have  $du = \cos x \, dx$ . Thus

$$\begin{aligned} \int \cos^3 x \, dx &= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cos x \, dx \\ &= \int (1 - u^2) \, du = u - \frac{1}{3}u^3 + C \\ &= \sin x - \frac{1}{3}\sin^3 x + C \end{aligned}$$

The method used in Example 1 suggests the following general strategy to be used in evaluating integrals of the form  $\int \sin^m x \cos^n x \, dx$ , where  $m \geq 0$  and  $n \geq 0$  are integers and either  $m$  or  $n$  is odd.

### HOW TO EVALUATE $\int \sin^m x \cos^n x \, dx$

- (a) If the power of cosine is odd ( $n = 2k + 1$ ), save one cosine factor and use  $\cos^2 x = 1 - \sin^2 x$  to express the remaining factors in terms of sine:

$$\begin{aligned} \int \sin^m x \cos^{2k+1} x \, dx &= \int \sin^m x (\cos^2 x)^k \cos x \, dx \\ &= \int \sin^m x (1 - \sin^2 x)^k \cos x \, dx \end{aligned}$$

Then substitute  $u = \sin x$ .

- (b) If the power of sine is odd ( $m = 2k + 1$ ), save one sine factor and use  $\sin^2 x = 1 - \cos^2 x$  to express the remaining factors in terms of cosine:

$$\begin{aligned} \int \sin^{2k+1} x \cos^n x \, dx &= \int (\sin^2 x)^k \cos^n x \sin x \, dx \\ &= \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx \end{aligned}$$

Then substitute  $u = \cos x$ .

Figure 1 shows the graphs of the integrand  $\sin^2 x \cos^2 x$  in Example 2 and its indefinite integral (with  $C = 0$ ). Which is which?

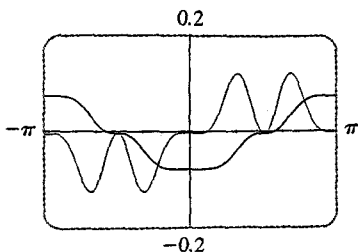


FIGURE 1

Example 3 shows that the area of the region shown in Figure 2 is  $\pi/2$ .

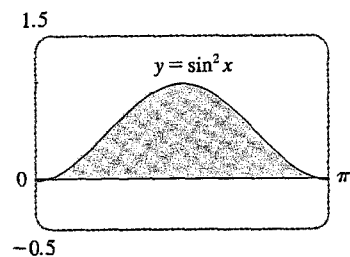


FIGURE 2

**EXAMPLE 2** Find  $\int \sin^5 x \cos^2 x \, dx$ .

**SOLUTION** Here the power of sine is odd and so we proceed as in case (b), substituting  $u = \cos x$ :

$$\begin{aligned} \int \sin^5 x \cos^2 x \, dx &= \int \sin^4 x \cos^2 x \sin x \, dx \\ &= \int (1 - \cos^2 x)^2 \cos^2 x \sin x \, dx \\ &= \int (1 - u^2)^2 u^2 (-du) = -\int (u^2 - 2u^4 + u^6) \, du \\ &= -\left(\frac{u^3}{3} - 2\frac{u^5}{5} + \frac{u^7}{7}\right) + C \\ &= -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C \end{aligned}$$

The advice in (a) and (b) works if either sine or cosine has an odd exponent. In the remaining case (both  $m$  and  $n$  are even), we proceed as follows.

(c) If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

It is sometimes helpful to use the identity

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

**EXAMPLE 3** Evaluate  $\int_0^\pi \sin^2 x \, dx$ .

**SOLUTION** Here  $m = 2$  and  $n = 0$ , so we use the half-angle formula for  $\sin^2 x$ :

$$\begin{aligned} \int_0^\pi \sin^2 x \, dx &= \frac{1}{2} \int_0^\pi (1 - \cos 2x) \, dx = \left[\frac{1}{2}(x - \frac{1}{2}\sin 2x)\right]_0^\pi \\ &= \frac{1}{2}(\pi - \frac{1}{2}\sin 2\pi) - \frac{1}{2}(0 - \frac{1}{2}\sin 0) = \frac{1}{2}\pi \end{aligned}$$

Notice that we mentally made the substitution  $u = 2x$  when integrating  $\cos 2x$ . Another method for evaluating this integral was given in Exercise 37 in Section 7.1.

**EXAMPLE 4** Find  $\int \sin^4 x \, dx$ .

**SOLUTION** It is possible to evaluate this integral using the reduction formula for  $\int \sin^n x \, dx$  (Equation 7.1.7) together with Example 1 (as in Exercise 37 in Section 7.1), but another method is to write  $\sin^4 x = (\sin^2 x)^2$  and use (c):

$$\begin{aligned} \int \sin^4 x \, dx &= \int (\sin^2 x)^2 \, dx \\ &= \int \left(\frac{1 - \cos 2x}{2}\right)^2 \, dx \\ &= \frac{1}{4} \int (1 - 2\cos 2x + \cos^2 2x) \, dx \end{aligned}$$

Since  $\cos^2 2x$  occurs, we must use another half-angle formula

$$\cos^2 2x = \frac{1}{2}(1 + \cos 4x)$$

This gives

$$\begin{aligned} \int \sin^4 x \, dx &= \frac{1}{4} \int [1 - 2 \cos 2x + \frac{1}{2}(1 + \cos 4x)] \, dx \\ &= \frac{1}{4} \int (\frac{3}{2} - 2 \cos 2x + \frac{1}{2} \cos 4x) \, dx \\ &= \frac{1}{4} (\frac{3}{2}x - \sin 2x + \frac{1}{8} \sin 4x) + C \end{aligned}$$

Integrals of the form  $\int \tan^m x \sec^n x \, dx$  can be integrated in the following cases.

#### HOW TO EVALUATE $\int \tan^m x \sec^n x \, dx$

(a) If the power of secant is even ( $n = 2k$ ), save a factor of  $\sec^2 x$  and use  $\sec^2 x = 1 + \tan^2 x$  to express the remaining factors in terms of  $\tan x$ :

$$\begin{aligned} \int \tan^m x \sec^{2k} x \, dx &= \int \tan^m x (\sec^2 x)^{k-1} \sec^2 x \, dx \\ &= \int \tan^m x (1 + \tan^2 x)^{k-1} \sec^2 x \, dx \end{aligned}$$

Then substitute  $u = \tan x$ .

(b) If the power of tangent is odd ( $m = 2k + 1$ ), save a factor of  $\sec x \tan x$  and use  $\tan^2 x = \sec^2 x - 1$  to express the remaining factors in terms of  $\sec x$ :

$$\begin{aligned} \int \tan^{2k+1} x \sec^n x \, dx &= \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x \, dx \\ &= \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x \, dx \end{aligned}$$

Then substitute  $u = \sec x$ .

**EXAMPLE 5** Evaluate  $\int \tan^6 x \sec^4 x \, dx$ .

**SOLUTION** Since the secant has an even exponent, we factor  $\sec^2 x$  from the integrand and substitute  $u = \tan x$  so that  $du = \sec^2 x \, dx$ . The rest of the integrand is then expressed completely in terms of  $\tan x$  by means of the identity  $\sec^2 x = 1 + \tan^2 x$ :

$$\begin{aligned} \int \tan^6 x \sec^4 x \, dx &= \int \tan^6 x \sec^2 x \sec^2 x \, dx \\ &= \int \tan^6 x (1 + \tan^2 x) \sec^2 x \, dx \\ &= \int u^6 (1 + u^2) \, du = \int (u^6 + u^8) \, du \\ &= \frac{u^7}{7} + \frac{u^9}{9} + C \\ &= \frac{1}{7} \tan^7 x + \frac{1}{9} \tan^9 x + C \end{aligned}$$

**EXAMPLE 6** Find  $\int \tan^5 x \sec^7 x dx$ .

**SOLUTION** Since the power of  $\tan x$  is odd, we factor  $\tan x \sec x$  from the integrand and substitute  $u = \sec x$  so that  $du = \sec x \tan x dx$ . Since an even power of  $\tan x$  remains, we use the identity  $\tan^2 x = \sec^2 x - 1$  to express the remainder of the integral completely in terms of  $\sec x$ :

$$\begin{aligned} \int \tan^5 x \sec^7 x dx &= \int \tan^4 x \sec^6 x \sec x \tan x dx \\ &= \int (\sec^2 x - 1)^2 \sec^6 x \sec x \tan x dx \\ &= \int (u^2 - 1)^2 u^6 du = \int (u^{10} - 2u^8 + u^6) du \\ &= \frac{u^{11}}{11} - 2\frac{u^9}{9} + \frac{u^7}{7} + C \\ &= \frac{1}{11} \sec^{11} x - \frac{2}{9} \sec^9 x + \frac{1}{7} \sec^7 x + C \quad \blacksquare \end{aligned}$$

If  $n = 0$ , only  $\tan x$  occurs. Here we use  $\tan^2 x = \sec^2 x - 1$  and, if necessary, the formula

$$\int \tan x dx = \ln |\sec x| + C$$

**EXAMPLE 7** Find  $\int \tan^3 x dx$ .

**SOLUTION**

$$\begin{aligned} \int \tan^3 x dx &= \int \tan x \tan^2 x dx \\ &= \int \tan x (\sec^2 x - 1) dx \\ &= \int \tan x \sec^2 x dx - \int \tan x dx \\ &= \frac{\tan^2 x}{2} - \ln |\sec x| + C \end{aligned}$$

In the first integral we mentally substituted  $u = \tan x$  so that  $du = \sec^2 x dx$ .  $\blacksquare$

If  $n$  is odd and  $m$  is even, we express the integrand completely in terms of  $\sec x$ . Powers of  $\sec x$  may require integration by parts.

**EXAMPLE 8** Find  $\int \sec x dx$ .

**SOLUTION** We multiply numerator and denominator by  $\sec x + \tan x$ :

$$\begin{aligned} \int \sec x dx &= \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx \\ &= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx \end{aligned}$$

If we substitute  $u = \sec x + \tan x$ , then  $du = (\sec x \tan x + \sec^2 x) dx$ , so the integral

becomes  $\int (1/u) du = \ln|u| + C$ . Thus we have

$$(1) \quad \int \sec x dx = \ln|\sec x + \tan x| + C$$

The method of Example 8 was admittedly very tricky, but we need Formula 1 for our future work.

**EXAMPLE 9** Find  $\int \sec^3 x dx$ .

**SOLUTION** Here we integrate by parts with

$$u = \sec x \qquad dv = \sec^2 x dx$$

$$du = \sec x \tan x dx \qquad v = \tan x$$

$$\begin{aligned} \int \sec^3 x dx &= \sec x \tan x - \int \sec x \tan^2 x dx \\ &= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx \\ &= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx \end{aligned}$$

Using Formula 1 and solving for the required integral, we get

$$\int \sec^3 x dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$$

Integrals such as the one in Example 9 may seem very special but they occur frequently in applications of integration, as we will see in Chapter 8. Integrals of the form  $\int \cot^m x \csc^n x dx$  can be found by similar methods because of the identity  $1 + \cot^2 x = \csc^2 x$ .

(2) To evaluate the integrals (a)  $\int \sin mx \cos nx dx$ , (b)  $\int \sin mx \sin nx dx$ , or (c)  $\int \cos mx \cos nx dx$ , use the corresponding identity:

$$(a) \quad \sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$(b) \quad \sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$(c) \quad \cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

**EXAMPLE 10** Evaluate  $\int \sin 4x \cos 5x dx$ .

**SOLUTION** This integral could be evaluated using integration by parts, but it is easier to use the identity in Equation 2(a) as follows:

$$\begin{aligned} \int \sin 4x \cos 5x dx &= \int \frac{1}{2}[\sin(-x) + \sin 9x] dx \\ &= \frac{1}{2} \int (-\sin x + \sin 9x) dx \\ &= \frac{1}{2}(\cos x - \frac{1}{9} \cos 9x) + C \end{aligned}$$

## EXERCISES 7.2

1–44 ■ Evaluate the integral.

1.  $\int_0^{\pi/2} \sin^2 3x \, dx$
2.  $\int_0^{\pi/2} \cos^2 x \, dx$
3.  $\int \cos^4 x \, dx$
4.  $\int \sin^3 x \, dx$
5.  $\int \sin^3 x \cos^4 x \, dx$
6.  $\int \sin^4 x \cos^3 x \, dx$
7.  $\int_0^{\pi/4} \sin^4 x \cos^2 x \, dx$
8.  $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$
9.  $\int (1 - \sin 2x)^2 \, dx$
10.  $\int \sin\left(x + \frac{\pi}{6}\right) \cos x \, dx$
11.  $\int \cos^5 x \sin^5 x \, dx$
12.  $\int \sin^6 x \, dx$
13.  $\int \sin^3 x \sqrt{\cos x} \, dx$
14.  $\int x \sin^3(x^2) \, dx$
15.  $\int \cos^2 x \tan^3 x \, dx$
16.  $\int \cot^3 x \sin^2 x \, dx$
17.  $\int \frac{1 - \sin x}{\cos x} \, dx$
18.  $\int \frac{dx}{1 - \sin x}$
19.  $\int \tan^2 x \, dx$
20.  $\int \tan^4 x \, dx$
21.  $\int \sec^4 x \, dx$
22.  $\int \sec^6 x \, dx$
23.  $\int_0^{\pi/4} \tan^4 x \sec^2 x \, dx$
24.  $\int_0^{\pi/4} \tan^2 x \sec^4 x \, dx$
25.  $\int \tan x \sec^3 x \, dx$
26.  $\int \tan^3 x \sec^3 x \, dx$
27.  $\int \tan^5 x \, dx$
28.  $\int \tan^6 x \, dx$
29.  $\int_0^{\pi/3} \tan^5 x \sec x \, dx$
30.  $\int_0^{\pi/3} \tan^3 x \sec^3 x \, dx$
31.  $\int \frac{\sec^2 x}{\cot x} \, dx$
32.  $\int \tan^2 x \sec x \, dx$
33.  $\int_{\pi/6}^{\pi/2} \cot^2 x \, dx$
34.  $\int_{\pi/4}^{\pi/2} \cot^3 x \, dx$
35.  $\int \cot^4 x \csc^4 x \, dx$
36.  $\int \cot^3 x \csc^4 x \, dx$
37.  $\int \csc x \, dx$
38.  $\int \csc^3 x \, dx$
39.  $\int \sin 5x \sin 2x \, dx$
40.  $\int \sin 3x \cos x \, dx$
41.  $\int \cos 3x \cos 4x \, dx$
42.  $\int \sin 3x \sin 6x \, dx$
43.  $\int \frac{1 - \tan^2 x}{\sec^2 x} \, dx$
44.  $\int \frac{\cos x + \sin x}{\sin 2x} \, dx$

45–46 ■ Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the integrand and its antiderivative (taking  $C = 0$ ).

45.  $\int \sin^5 x \, dx$
46.  $\int \sin^4 x \cos^4 x \, dx$

47. Find the average value of the function  $f(x) = \sin^2 x \cos^3 x$  on the interval  $[-\pi, \pi]$ .

48. Evaluate  $\int \sin x \cos x \, dx$  by four methods: (a) the substitution  $u = \cos x$ , (b) the substitution  $u = \sin x$ , (c) the identity  $\sin 2x = 2 \sin x \cos x$ , and (d) integration by parts. Explain the different appearances of the answers.

49–50 ■ Find the area of the region bounded by the given curves.

49.  $y = \sin x$ ,  $y = \sin^3 x$ ,  $x = 0$ ,  $x = \pi/2$
50.  $y = \sin x$ ,  $y = 2 \sin^2 x$ ,  $x = 0$ ,  $x = \pi/2$

51–52 ■ Use a graph of the integrand to guess the value of the integral. Then use the methods of this section to prove that your guess is correct.

51.  $\int_0^{2\pi} \cos^3 x \, dx$
52.  $\int_0^2 \sin 2\pi x \cos 5\pi x \, dx$

53–56 ■ Find the volume obtained by rotating the region bounded by the given curves about the specified axis.

53.  $y = \sin x$ ,  $x = \pi/2$ ,  $x = \pi$ ,  $y = 0$ ; about the  $x$ -axis
54.  $y = \tan^2 x$ ,  $y = 0$ ,  $x = 0$ ,  $x = \pi/4$ ; about the  $x$ -axis
55.  $y = \cos x$ ,  $y = 0$ ,  $x = 0$ ,  $x = \pi/2$ ; about  $y = -1$
56.  $y = \cos x$ ,  $y = 0$ ,  $x = 0$ ,  $x = \pi/2$ ; about  $y = 1$

57. A particle moves on a straight line with velocity function  $v(t) = \sin \omega t \cos^2 \omega t$ . Find its position function  $s = f(t)$  if  $f(0) = 0$ .

58. Household electricity is supplied in the form of alternating current that varies from 155 V to  $-155$  V with a frequency of 60 cycles per second (Hz). The voltage is thus given by the equation

$$E(t) = 155 \sin(120 \pi t)$$

where  $t$  is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of  $[E(t)]^2$  over one cycle. Calculate the RMS voltage of household current.

11.  $16\pi/3$     13.  $4\pi(2ah + h^2)^{3/2}/3$   
 15.  $\int_0^1 \pi[(1-x^2)^2 - (1-x^2)] dx$   
 17. (a)  $2\pi/15$     (b)  $\pi/6$     (c)  $8\pi/15$   
 19. (a) 0.38    (b) 0.87  
 21. Solid obtained by rotating the region under  $y = \sin x$  from 0 to  $\pi$  about the  $x$ -axis  
 23. Solid obtained by rotating the region in the first quadrant bounded by  $x = 4 - y^2$  and the axes about the  $x$ -axis  
 25. 36    27.  $125\sqrt{3}/3 \text{ m}^3$     29. 3.2 J  
 31. (a)  $8000\pi/3 \text{ ft}\cdot\text{lb}$     (b) 2.1 ft    33.  $f(x)$

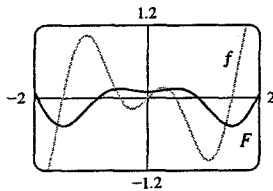
PROBLEMS PLUS ■ page 412

1.  $\pi/2$     5. 1    7. Does not exist  
 9.  $f(x) = \sqrt{2x/\pi}$     13.  $e^{-2}$     15.  $[-1, 2]$   
 17.  $1/\sqrt{2}$     19.  $\sqrt{1 + \sin^4 x} \cos x$     23.  $c = -1; \frac{1}{2}$   
 25.  $2x \int_0^x \sin(t^2) dt + x^2 \sin(x^2)$     27.  $y = \frac{32}{9}x^2$   
 31.  $\pi/(30\sqrt{2})$

CHAPTER 7

Exercises 7.1 ■ page 421

1.  $(xe^{2x}/2) - (e^{2x}/4) + C$     3.  $-\frac{1}{4}x \cos 4x + \frac{1}{16} \sin 4x + C$   
 5.  $\frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x - \frac{2}{27} \sin 3x + C$   
 7.  $x(\ln x)^2 - 2x \ln x + 2x + C$   
 9.  $\frac{1}{8}(\sin 2\theta - 2\theta \cos 2\theta) + C$     11.  $t^3(3 \ln t - 1)/9 + C$   
 13.  $e^{2\theta}(2 \sin 3\theta - 3 \cos 3\theta)/13 + C$   
 15.  $y \cosh y - \sinh y + C$     17.  $1 - 2/e$   
 19.  $-\frac{1}{2}$     21.  $(\pi + 6 - 3\sqrt{3})/6$   
 23.  $\sin x (\ln \sin x - 1) + C$     25.  $(2x + 1)e^x + C$   
 27.  $x(\cos \ln x + \sin \ln x)/2 + C$     29.  $4 \ln 2 - \frac{3}{2}$   
 31.  $2(\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}) + C$   
 33.  $e^{x^2}[(x^4/2) - x^2 + 1] + C$   
 35.  $(x \sin \pi x)/\pi + (\cos \pi x)/\pi^2 + C$

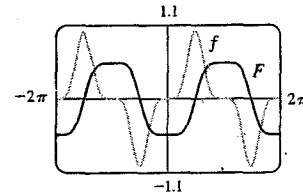


37. (b)  $-\frac{1}{4} \cos x \sin^3 x + (3x/8) - \frac{3}{16} \sin 2x + C$   
 39. (b)  $\frac{2}{3}, \frac{8}{15}$     45.  $x[(\ln x)^3 - 3(\ln x)^2 + 6 \ln x - 6] + C$   
 47.  $(\pi + 6\sqrt{3} - 12)/12$     49. 0, 0.70; 0.08  
 51.  $10\pi^2$     53.  $2\pi e$     55.  $2 - e^{-t}(t^2 + 2t + 2) \text{ m}$   
 59. 1

Exercises 7.2 ■ page 428

1.  $\pi/4$     3.  $\frac{3}{8}x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$   
 5.  $\frac{1}{2} \cos^7 x - \frac{1}{3} \cos^5 x + C$     7.  $(3\pi - 4)/192$

9.  $(3x/2) + \cos 2x - \frac{1}{8} \sin 4x + C$   
 11.  $\frac{1}{6} \sin^6 x - \frac{1}{4} \sin^8 x + \frac{1}{10} \sin^{10} x + C$   
 [or  $-\frac{1}{6} \cos^6 x + \frac{1}{4} \cos^8 x - \frac{1}{10} \cos^{10} x + C_1$ ]  
 13.  $[\frac{2}{7} \cos^3 x - \frac{2}{3} \cos x] \sqrt{\cos x} + C$   
 15.  $\frac{1}{2} \cos^2 x - \ln |\cos x| + C$     17.  $\ln(1 + \sin x) + C$   
 19.  $\tan x - x + C$     21.  $\tan x + \frac{1}{3} \tan^3 x + C$     23.  $\frac{1}{5}$   
 25.  $\frac{1}{3} \sec^3 x + C$     27.  $\frac{1}{4} \sec^4 x - \tan^2 x + \ln |\sec x| + C$   
 29.  $\frac{38}{15}$     31.  $\frac{1}{2} \tan^2 x + C$     33.  $\sqrt{3} - (\pi/3)$   
 35.  $-\frac{1}{3} \cot^3 x - \frac{1}{7} \cot^7 x + C$     37.  $\ln |\csc x - \cot x| + C$   
 39.  $\frac{1}{2}[\frac{1}{3} \sin 3x - \frac{1}{7} \sin 7x] + C$     41.  $\frac{1}{2}[\sin x + \frac{1}{3} \sin 3x] + C$   
 43.  $\frac{1}{2} \sin 2x + C$   
 45.  $-\frac{1}{3} \cos^5 x + \frac{2}{3} \cos^3 x - \cos x + C$



47. 0    49.  $\frac{1}{3}$     51. 0    53.  $\pi^2/4$     55.  $2\pi + (\pi^2/4)$   
 57.  $s = (1 - \cos^3 \omega t)/(3\omega)$

Exercises 7.3 ■ page 434

1.  $2/\sqrt{3}$     3.  $-\sqrt{1-x^2} + C$   
 5.  $\frac{1}{4} \sin^{-1}(2x) + \frac{1}{2} x \sqrt{1-4x^2} + C$     7.  $\ln(1 + \sqrt{2})$   
 9.  $[\sqrt{x^2 - 16}/(32x^2)] + \frac{1}{128} \sec^{-1}(x/4) + C$   
 11.  $\sqrt{9x^2 - 4} - 2 \sec^{-1}(3x/2) + C$   
 13.  $(x/\sqrt{a^2 - x^2}) - \sin^{-1}(x/a) + C$   
 15.  $(1/\sqrt{3}) \ln |(\sqrt{x^2 + 3} - \sqrt{3})/x| + C$   
 17.  $\frac{64}{1215}$     19.  $\frac{5}{3}(1 + x^2)^{3/2} + C$   
 21.  $\frac{1}{2}[\sin^{-1}(x-1) + (x-1)\sqrt{2x-x^2}] + C$   
 23.  $\frac{1}{3} \ln |3x + 1 + \sqrt{9x^2 + 6x - 8}| + C$   
 25.  $\frac{1}{2}[\tan^{-1}(x+1) + (x+1)/(x^2 + 2x + 2)] + C$   
 27.  $\frac{1}{2}[e^{\sqrt{9 - e^{2t}}} + 9 \sin^{-1}(e^t/3)] + C$     33. 0.81, 2; 2.10  
 35.  $r\sqrt{R^2 - r^2} + \pi r^2/2 - R^2 \arcsin(r/R)$     37.  $2\pi^2 R r^2$

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1.  $\frac{A}{x-1} + \frac{B}{x+2}$     3.  $\frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{2x+3}$   
 5.  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}$     7.  $1 + \frac{A}{x-1} + \frac{B}{x+1}$   
 9.  $\frac{A}{x} + \frac{Bx+C}{x^2+2}$     11.  $\frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2}$   
 13.  $\frac{Ax+B}{x^2+9} + \frac{Cx+D}{(x^2+9)^2} + \frac{Ex+F}{(x^2+9)^3}$   
 15.  $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+x+2}$     17.  $\frac{x^2}{2} - x + \ln|x+1| + C$   
 19.  $\ln 3 + 3 \ln 6 - 3 \ln 4 = \ln \frac{81}{8}$   
 21.  $3x - 7 \ln |2x + 3| + C$   
 23.  $x - \ln|x| + 2 \ln|x-1| + C = x + \ln \frac{(x-1)^2}{|x|} + C$   
 25.  $2 \ln 2 + \frac{1}{2}$     27.  $4 \ln 6 - 3 \ln 5$