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length of curve in a plane

$$s = \int_{t=a}^{t=b} \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}\, dt, \quad x = f(t), \; y = g(t)$$

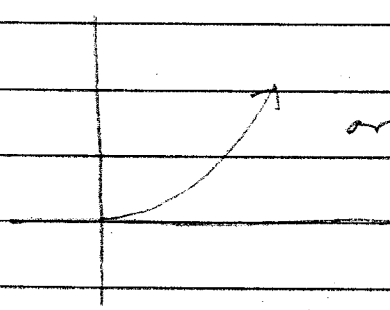
$y = x^2$? view x as the parameter

$t \in \mathbb{R}$ $\quad y = t^2, \; x = t$

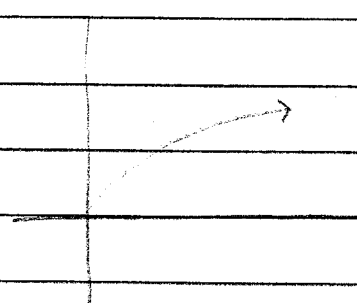
$$s = \int_a^b \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx$$

Ex: $y = x^{\frac{3}{2}}$ from $(1,1)$ to $(4,8)$

Get length



or



Which?

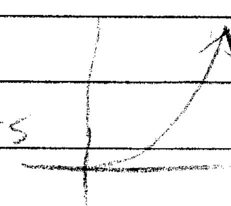
find sign of 2nd deriv. if +, concave up

if − concave down

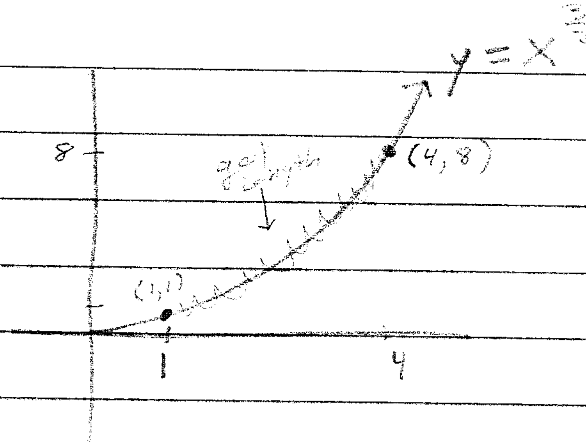
$y' = \frac{3}{2}x^{\frac{1}{2}}$

$y'' = \frac{3}{4}x^{-\frac{1}{2}}$

$= \frac{3}{4\sqrt{x}}$ looks pos! so its



So



parametric eqns: $y = t^{\frac{3}{2}}$

$x = t$

limits of integration (since (1,1) + (4,8) are x + y values)

well, when $x = 1$, $t = 1$

when $x = 4$, $t = 4$

so $1 \le t \le 4$

$$s = \int_1^4 \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}\ dt$$

$$= \int_1^4 \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\ dx$$

$$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$$

$$= \int_1^4 \sqrt{1 + \frac{9}{4}x}\ dx$$

$u = 1 + \frac{9}{4}x$ so $du = \frac{9}{4} dx$

two ways to do this,

we'll do indef. integral way.

$u = 1 + \frac{9}{4}x$

$du = \frac{9}{4}\,dx$

$\frac{4}{9}du = dx$

so indef. integral is $\frac{4}{9}\int u^{\frac{1}{2}}\,du$

$= \frac{4}{9}\left[\frac{2}{3}u^{\frac{3}{2}}\right]$

back to the def. integral.

$$= \frac{4}{9}\left[\frac{2}{3}\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{27}\left[10^{\frac{3}{2}} - \left(\frac{13}{4}\right)^{\frac{3}{2}}\right]$$

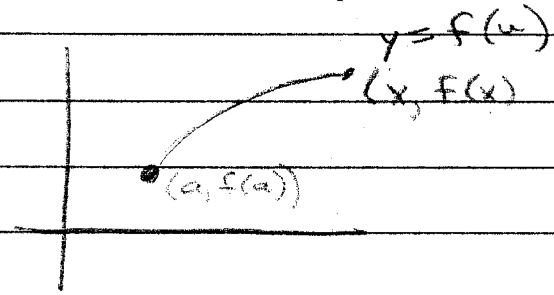
That's it; pretty ugly though.

differential of arc length

$y = f(u)$

$$s(x) = \int_a^x \sqrt{1 + \left[\frac{dy}{du}\right]^2}\, du$$

$$= \int_a^x \sqrt{1 + [f'(u)]^2}\, du \qquad * \text{ a fn of } x$$



$$s'(x) = \frac{ds}{dx} = \frac{d}{dx} \int_a^x \sqrt{1 + [f'(u)]^2}\, du$$

$$\frac{ds}{dx} = \sqrt{1 + [f'(x)]^2}$$

x came from Fund. Thm of Calculus

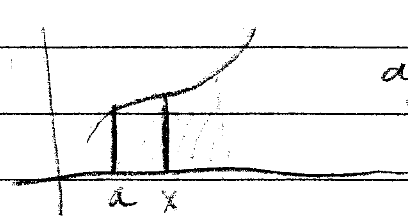
$$ds = \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx$$

Really, so where did the x come from?

* above,

then what?

consider what an integral is



area under curve depends on x. As x moves along, area under curve changes.

as $x \rightarrow$ we accumulate area under curve,
so essentially we're doing this!

$$D_x \int_a^x f(t)\,dt$$

$\underbrace{\int_a^x f(t)\,dt}$ Area accumulate under f(t) as t goes from a to x.

t has ϕ to do w/any parameters

The instantaneous rate of change of $y = g(x)$ at $x = c$.

start here & become familiar w/ the FTC.