9/11/08

HW: 13, 15, 17 [6.4]

Wittgenstein, Logical Positivism, Vienna Circle

Can't be a private language - have to be able to use it improperly. (Wittgenstein)

Re Noah's statement about intuition. (how do you know when your intuition is wrong?) {when all mathematics crumbles, you look to your intuition ... <LATER> ... Example w/ intuition WRONG}

So is intuition a private lang in math?

and Re Area is figment of imagination... just like everything else in math... until you define it.

or

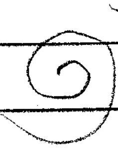
Area is real

2 views...

QUOTED from a Book on Analysis NOT Noah.

OK, now some math...

Arc length (review from last time)

 definitely has length & we ~~ought to be able to compute it.~~

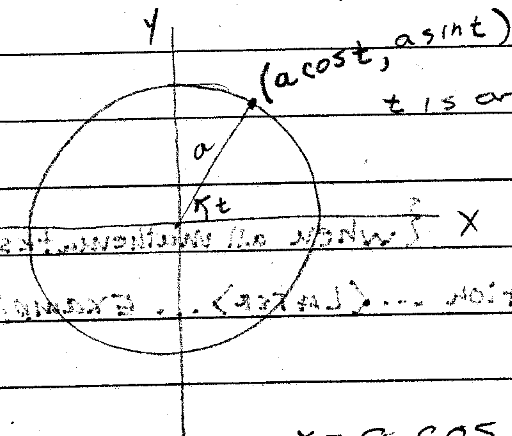
But it isn't a fn.

But can we find some indep variable & have it be a fn of that

Parameter,

Parametric eqns

Consider a circle, radius a



t is arc length, since radians

$x = a\cos t, \quad y = a\sin t$

these are the parametric eqns of the circle, parameter t.

Now x & y are fns of t.

As values of t increase from 0 to 2π,

$0 \le t \le 2\pi$

the dot is tracing a circle

So spiral:

$x = t\cos t$

$y = t\sin t$

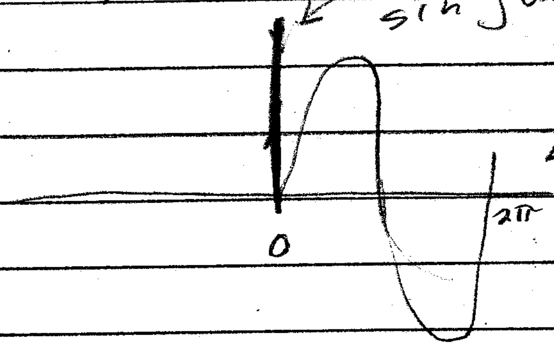
let's have $0 \le t \le 5\pi$

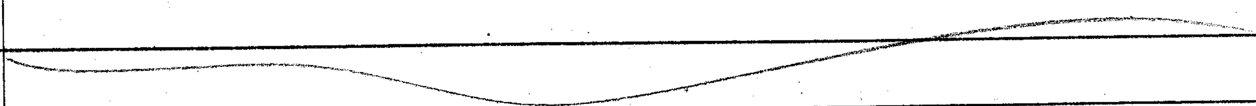
as t increases, so does radius of circle sine fn

$y = \sin t$

$x = t$

sin just goes up & down & x moves





$x = 2t + 1$

$y = t^2 - 1$

$0 \le t \le 3$

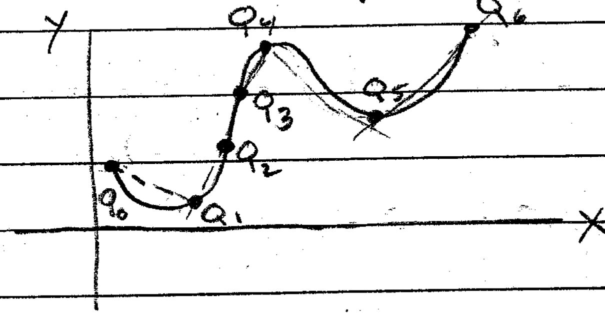
Graph

t	x	y
0	1	-1
1	3	0
2	etc	

substitute values & plot the pts

it's directed

Find length of this:



consider it parametrically: x coord is some fn of t, so is y.

$x = f(t)$

$y = g(t)$

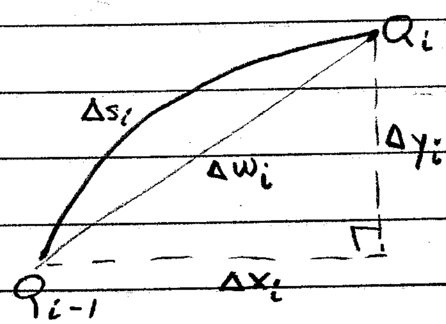
$t \in [a, b]$

So $a = t_0, t_1, t_2, \ldots, t_{i-1}, t_i \ldots t_n = b$

partitions, need not be same length

see points on graph

length of arc $\approx$ sum of segments; more accurate as we make norm of partition go to zero.



then we see Pythag. Thm

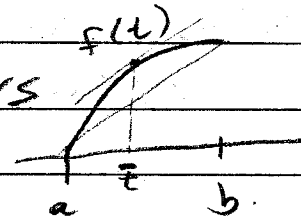
So $\Delta x_i = f(t_i) - f(t_{i-1})$

and $\Delta y_i = g(t_i) - g(t_{i-1})$

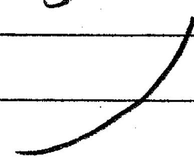
and then $\Delta w_i = \sqrt{[f(t_i) - f(t_{i-1})]^2 + [g(t_i) - g(t_{i-1})]^2}$

All that looks like mean value thm!

If fn f is doing this



and g this



MVT $\quad f(t_i) - f(t_{i-1}) = f'(\bar{t}_i)\Delta t_i$

and same for g: $g(t_i) - g(t_{i-1}) = g'(\hat{t}_i)\Delta t_i$

The Plan: $\sum_{i=1}^{n} \sqrt{[f'(\bar{t}_i)]^2 + [g'(\hat{t}_i)]^2}\ \Delta t_i$

why is it ok to ignore that the $\hat{t}$ and $\bar{t}$ are different? There is a thm.

$$\lim_{\|P\|\to 0} \sum_{i=1}^{n} \sqrt{[f'(t_i)]^2 + [g'(t_i)]^2}\ \Delta t_i$$

$$= \int_{t=a}^{t=b} \sqrt{\text{that mess}\uparrow}\ dt$$

$$= \int_a^b \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}\ dt$$

Find:
circumf of circle, radius r

x coord is $r\cos t$
y coord is $r\sin t$
consider $0 \le t \le 2\pi$

Then $\frac{dx}{dt} = r\sin t$, square it

$\frac{dy}{dt} = -r\cos t$, square it

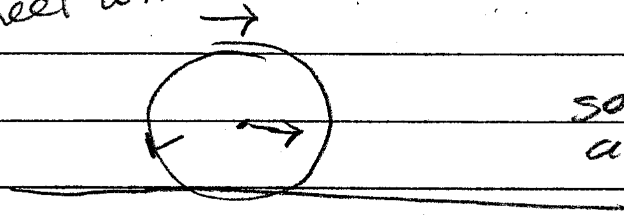
$$\text{circumf} = \int_0^{2\pi} \sqrt{r^2\sin^2 t + r^2\cos^2 t}\, dt$$

$$= r\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\, dt$$

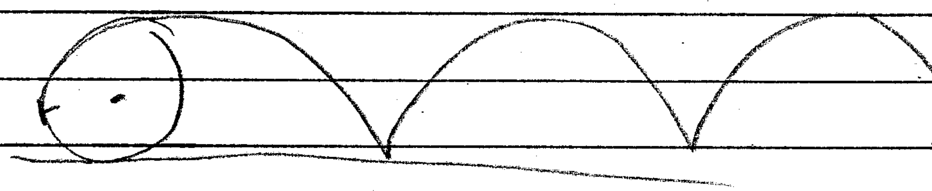
$$= r\int_0^{2\pi} dt$$

$$= r\,[t]_0^{2\pi} = 2\pi r - r(0) = 2\pi r$$

wheel with a tack in it,



something like
a sine curve



cycloid

or consider spokes,
with 2 flies on it

which
goes
farther