Length Plane Curve [6.4]

* Intro: Len one arch of cycloid.

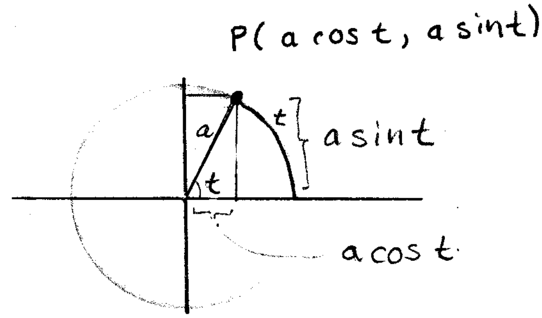
How long is this ?
It is not a fn of x or of y.
It's hard to see how to restrict Dmn to get fn.

Parametric Representation of curves

Circle

$x = a\cos t,\ y = a\sin t,\quad 0 \le t \le 2\pi$

Parametric eqns. of circle radius a.



Spiral

$x = t\cos t$
$y = t\sin t,\quad 0 \le t \le 5\pi$

Sine

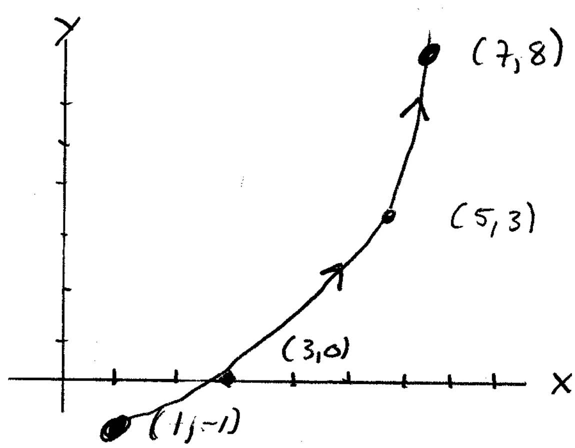
$y = \sin t$
$x = t,\quad 0 \le t \le \pi$

Parabola

$y = t$
$x = t^2,\quad -2 \le t \le 2$

GRAPH $x = 2t+1,\ y = t^2-1,\quad 0 \le t \le 3$

t	x	y
0	1	-1
1	3	0
2	5	3
3	7	8



Parametric curve has an orientation.

a **smooth** curve

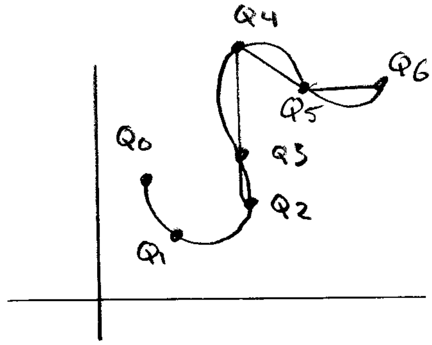
Def p294

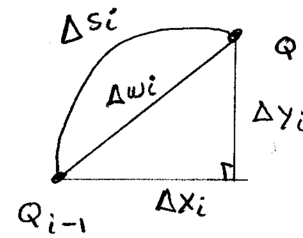
$x = f(t)$
$y = g(t),\quad a \le t \le b$

f', g' exist and continuous on $[a,b]$
$f'(t)$ and $g'(t)$ not both zero at same t on (a,b)

[6.4] Arc Len.

Arc Len





Partition

$a = t_0 < t_1 < t_2 < \dots < t_n = b$

Producing arcs w/ end pts $Q_0, Q_1, Q_2, \dots, Q_{n-1}, Q_n$

$$\Delta w_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$$\Delta w_i = \sqrt{[f(t_i) - f(t_{i-1})]^2 + [g(t_i) - g(t_{i-1})]^2}$$

By MVT

$$\left.\begin{aligned} f(t_i) - f(t_{i-1}) &= f'(\bar{t}_i)\,\Delta t_i \\ g(t_i) - g(t_{i-1}) &= g'(\hat{t}_i)\,\Delta t_i \end{aligned}\right\}$$

$\bar{t}_i, \hat{t}_i$ usually are not equal

$\Delta t = t_i - t_{i-1}$

$$\Delta w_i = \sqrt{[f'(\bar{t}_i)\Delta t_i]^2 + [g'(\hat{t}_i)\Delta t_i]^2}$$

$$\Delta w_i = \sqrt{[f'(\bar{t}_i)]^2 + [g'(\hat{t}_i)]^2}\ \Delta t_i$$

$$\sum_{i=1}^{n} \Delta w_i = \sum_{i=1}^{n} \sqrt{[f'(\bar{t}_i)]^2 + [g'(\hat{t}_i)]^2}\ \Delta t_i$$

Lim of above as norm of partition $\to 0$,

NOT exactly a Riemann sum, because $\bar{t}_i$ and $\hat{t}_i$ are not likely the same point.

But a Thm of analysis says that as Norm of partition goes to zero, this makes no difference.