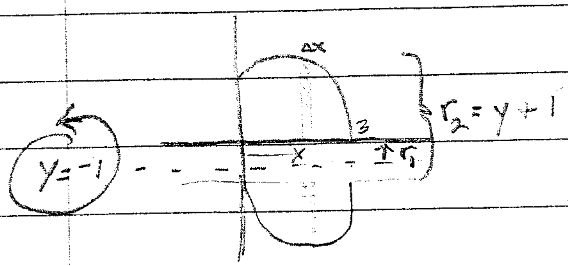
9/5 HW: From Putting it All Together
15, 17, 18, 20.

$y = 3 + 2x - x^2$, x-axis c) $y = -1$



disk, washer or shell?
washers

$\Delta V = \pi(r_2^2 - r_1^2)\Delta r$

$\to \Sigma$

outer radius, because rotated around y = -1, inner radius

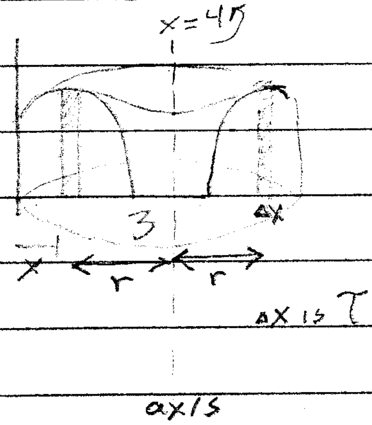
$$\Delta V = \pi\left[(3+2x-x^2+1)^2 - 1^2\right]\Delta x$$

$$= \pi\left[(4+2x-x^2)^2 - 1\right]\Delta x$$

$$V = \pi\int_0^3 (4+2x-x^2)^2 - 1\,dx$$

$$= \frac{243}{5}\pi$$

Now rotate it around $x=4$, everything else the same.



washers - integrating in y-direction.
highest pt of curve is at zero.
For radius, need x as fn of y.
Hard...!

Shells are better.

$\Delta V = 2\pi r h \tau$ ← thickness

radius w/respect to axis of rotation $r = 4 + x$ since distance is ? arbitrary.

Radius:
Try from other side
need r as some fn of x:

$x + r$ has to $= 4$

so $r = 4 - x$

Then we have

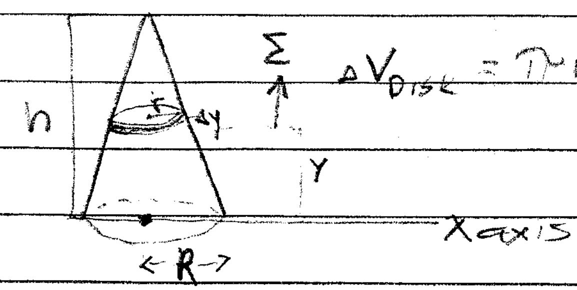
$$\Delta V = 2\pi r h \tau$$

$$= 2\pi(4-x)(3+2x-x^2)\,\Delta x$$

$$V = 2\pi \int_0^3 (4-x)(3+2x-x^2)\,dx$$

↑ the limits depend on the section!

Cone Problem - Derive Volume

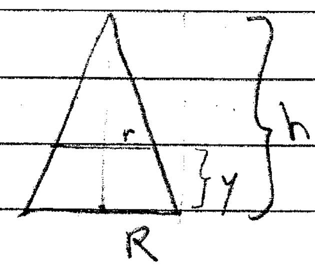


Σ $\uparrow$ $\Delta V_{DISK} = \pi r^2 \tau$

$r = g(y)$

cone radius = R

height h



similar triangles:

$$\frac{r}{R} = \frac{h-y}{h}$$

$$\Delta V = \pi \left[\frac{R}{h}(h-y) \right]^2 \Delta y$$

$$V_{cone} = \frac{R^2}{h^2} \pi \int_0^h (h-y)^2 \, dy$$

limits-
integrating $\uparrow$

$$V_{cone} = \frac{R^2}{h^2} \pi \left[h^2 y - h y^2 + \frac{1}{3} y^3 \right]_0^h$$

$$= \frac{R^2}{h^2} \pi \left[h^3 - h^3 + \frac{1}{3} h^3) - (0-0) \right]$$

$$= \frac{R^2}{h^2} \pi \frac{1}{3} h^3 = \frac{\pi R^2 h}{3}$$

$$= \frac{1}{3} \pi R^2 h$$