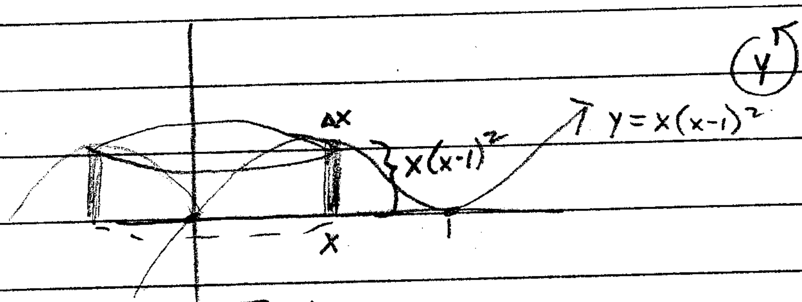
9/4

HW 6.3 15, 17, 18, 20

Shells, ctd

Region bounded by $y = x(x-1)^2$, and by the x-axis, rotated around y-axis.



summing the washers ↑ is very inconvenient. (hard to find the radii of the washer), and need to solve the cubic for y. Need another way. section it into cylindrical shell.

$$\Delta V = 2\pi r h$$

$$\Delta V = 2\pi x \cdot x(x-1)^2 \, \Delta x$$

exact $V = 2\pi \int_0^1 x^2(x-1)^2 \, dx$

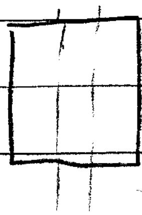
$$= 2\pi \int_0^1 x^4 - 2x^3 + x^2 \, dx$$

$$= 2\pi \left[\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{1}{3}x^3 \right]_0^1$$

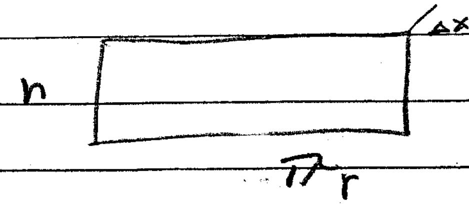
$$= 2\pi \left[\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right) - (0) \right]$$

$$= \frac{2\pi}{30} = \boxed{\frac{\pi}{15}}$$

derivation of V of hollow cyl.



hollow cylinder
cut & rolled out



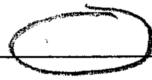
$$V = \pi r_2^2 h - \pi r_1^2 h$$

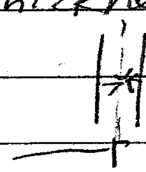
$$= \pi h (r_2^2 - r_1^2)$$

$$= \pi h (r_2 - r_1)(r_2 + r_1)$$

now multi. by $\frac{2}{2}$ (by tradition!)

$$= 2\pi h (r_2 - r_1) \frac{(r_2 + r_1)}{2}$$

 πr^2 thickness of wall → 0

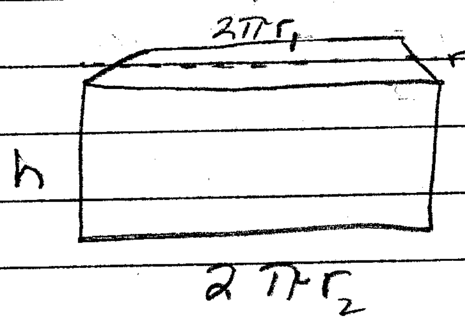


$$r = \frac{r_2 + r_1}{2}$$

$$r_2 - r_1 = \Delta r$$

So $V = 2\pi h r \Delta r$

so we'd have

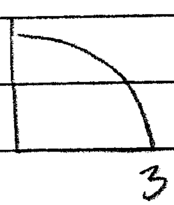


$2\pi r_1$ is shorter than $2\pi r_2$

Look at the section in Varberg, "Putting it all together" after shells.

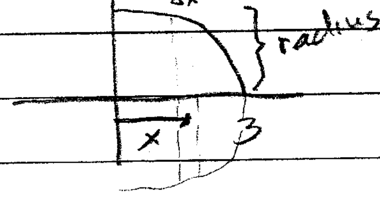
New probl $y = 3 + 2x - x^2$
x-axis, y-axis.

a) First rotate it around x axis, then
b) around y-axis, & then
c) around $y = -1$ and then
d) around $x = 4$



$y = 3 + 2x - x^2$

For a), rotate it around x axis,



use disk. slice vertically.

$V_{disk} = \pi r^2 \Delta r$

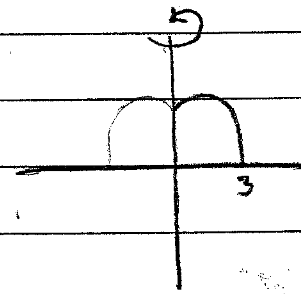
r is $3 + 2X - 2X^2$

$\Delta V = \pi r^2 \Delta r$

$\Delta V = \pi \left[3 + 2x - x^2\right]^2 \Delta x$

$V = \pi \int_0^3 (3+2x-x^2)^2\, dx$

$= \frac{153}{5}\pi$



b) now rotate it around y

hump or no hump?

Does it take max value at $x=0$?

Need to find where deriv is zero;

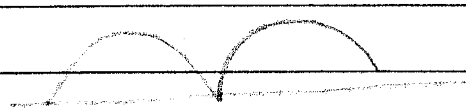
$2-2x=0$

$1-x=0$

$x=1$ so it has a hump

and we don't know where the inner radius is. Use shells instead of disks?

This one is like the shell prob earlier

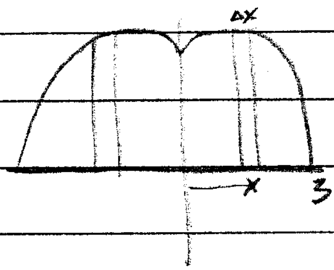


Does it matter whether the hole disappears or not?

disappears   disappears where fn ends

Note: Limits are 0 & 3. The limits of integ. depend on plane region, not on how it's rotated.

(b)



$$\Delta V_{shell} = 2\pi r h \Delta r$$

$$= 2\pi x(3+2x-x^2)\Delta x$$

$$= 2\pi \int_0^3 x(3+2x-x^2)\,dx$$

$$= \frac{45}{2}\pi$$