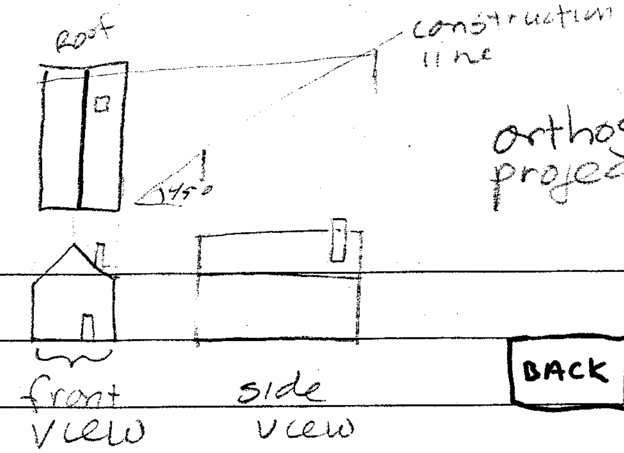
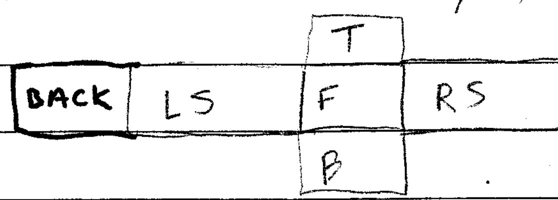
9/3



orthographic projection

Standard layout

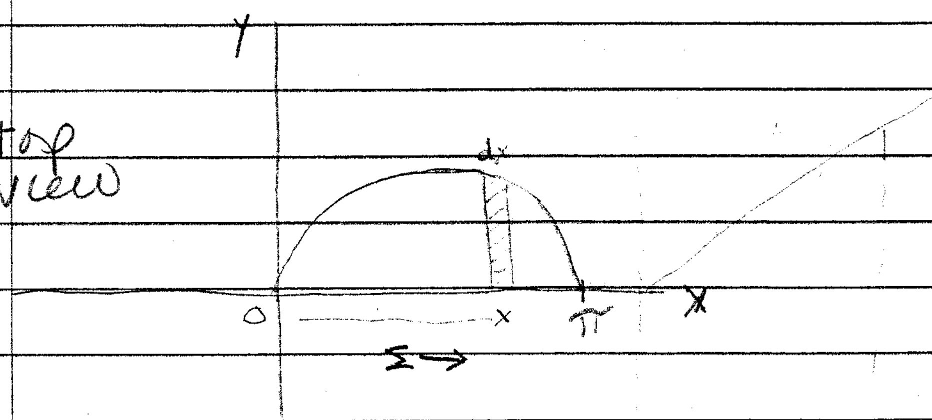


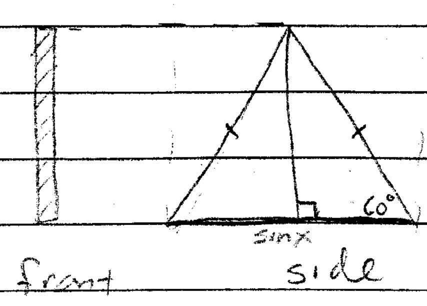
front + side views won't ~~tell you about~~ to other.

That in order to visualize this example

Base region one arch of $y = \sin x$ bounded by x-axis. Cross sections $\perp$ to x-axis are equilateral Δs. Find vol of the solid.

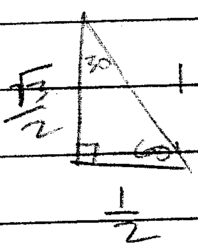
top view



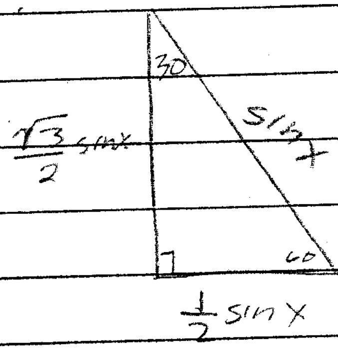


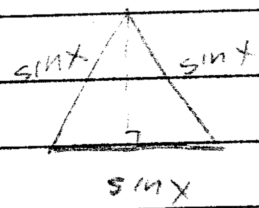
$\Delta V = \frac{1}{2}(\text{base})(\text{height})(\text{thickness})$

sinx is the length of the side of the triangle



30-60-90 triangle. so a scale drawing:





so $\Delta V = \frac{1}{2}(\text{base})(\text{height})(\text{thick})$

$= \frac{1}{2}(\sin x)\left(\frac{\sqrt{3}}{2}\sin x\right)\Delta X$

$V = \frac{\sqrt{3}}{4}\int_0^{\pi} \sin^2 x \, dx$

$= \frac{\sqrt{3}}{4}\int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$ nice identity ~~there~~

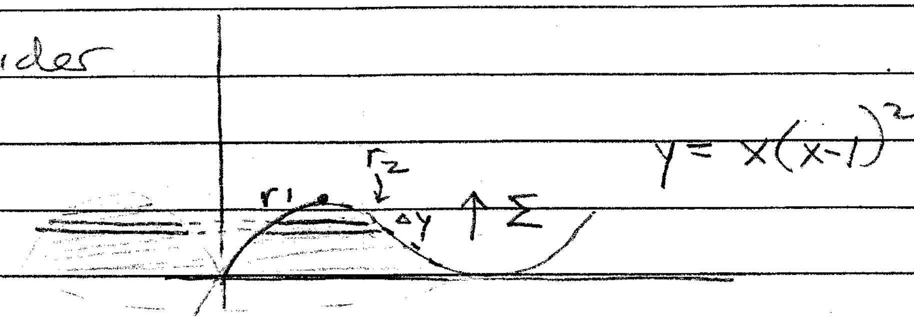
$= \frac{\sqrt{3}}{8}\int_0^{\pi} 1-\cos 2x \, dx$

$= \frac{\sqrt{3}}{8}\int_0^{\pi}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$

$= \frac{\sqrt{3}}{8}\left[(\pi-0)-(0-0)\right] = \frac{\sqrt{3}\pi}{8}$

Cylindrical shells method

Consider



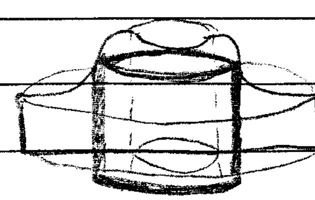
V solid formed by region bounded by $y = x(x-1)^2$ and the x-axis, y

Try washers... serious trouble because you need to know y_{max} of $x(x-1)^2$, which is a pain. Notice r_1 & r_2, the inner & outer radii of the washers. Hard.

So try a different idea:

Cylindrical shells

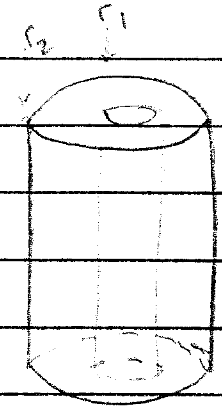
Consider the symmetric thing here →



inkwell

(also pictured in book) add up all the nested shells!

How to determine vol of shell



vol of cylinder outside − vol cylinder inside:

$$V_{cylinder} = \pi r_2^2 h - \pi r_1^2 h$$

$$= \pi h (r_2^2 - r_1^2)$$

$$= \pi h (r_2 - r_1)(r_2 + r_1)$$

mult by $\frac{2}{2}$: $$= 2\pi h \frac{r_2 + r_1}{2} (r_2 - r_1)$$

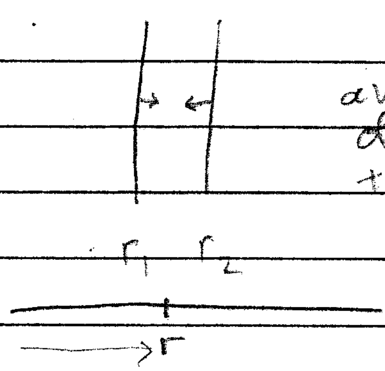
why? → $\frac{r_2 + r_1}{2} = r$ (average of radii)

and $r_2 - r_1 = \Delta r$

So $V_{cyl} = \underbrace{2\pi r}_{} \, h \, \Delta r$

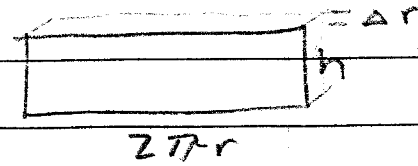
circumf × height × thickness

$\Delta r \to 0$



average of the two will go to r.

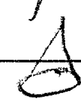
cut it like a tin can



solids not ~~rotated~~, for HW

HW: 6.2 23, 25, 26

and get V of right circ cone



and get V of right square base pyramid