8/28 HW: continue the probs from yesterday.

P1

[6.2] semicircular Region

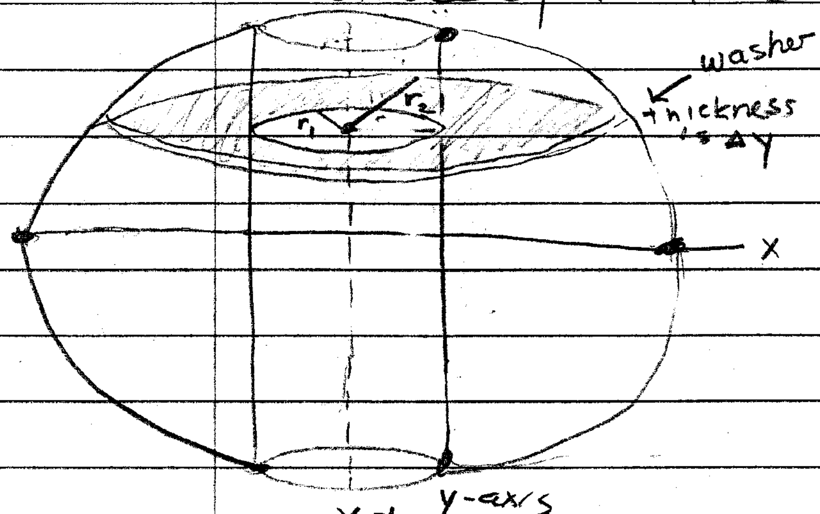
$x = \sqrt{4-y^2}$

$x=-1$

y-axis

$x = \sqrt{4-y^2}$ is $x^2 = 4-y^2$ is $x^2+y^2=2$

circle of radius 2, centered at (0,0)



This is not a sphere with a core taken out.

Divide it into large but finite # of washers + add their volumes. That's an approximation of the volume. Make it a better approx by using more washers infinitely many. The missed parts $\to 0$.

Consider the limit of that sum. If the limit exists, its the volume.

$\Delta V = \pi (r_2^2 - r_1^2) h$ h = thickness

outer radius is $1+\sqrt{4-y^2}$ (1 because distance from y axis to $x=-1$)

$\Delta V = \pi\left[(1+\sqrt{4-y^2})^2 - 1\right]\Delta y$

$\Delta V = \pi\left[1+2\sqrt{4-y^2}+4-y^2-1\right]\Delta y$

$V = \pi\int\left[4+2\sqrt{4-y^2}-y^2\right]dy$

limits of integration are -2 to 2

so $V = 2\pi \int_0^2 4 + 2\sqrt{4-y^2} - y^2 \, dy$

because it's symmetrical.

Now integrate that:

$2\pi \left[\int_0^2 4\,dy - \int_0^2 y^2 dy + \int_0^2 2\sqrt{4-y^2}\, dy \right]$

can integrate

can integrate

see p35 for how to integrate this. It's area of a quarter circle.

Prelude: $\int \sqrt{4-y^2}\, dy$

$y = 2\sin\theta$

$dy = 2\cos\theta \, d\theta$

$\int \sqrt{4 - 4\sin^2\theta}\,(2\cos\theta)\, d\theta$

$2\int \cos^2\theta \, d\theta$

and then use trig identities to integrate that thing!

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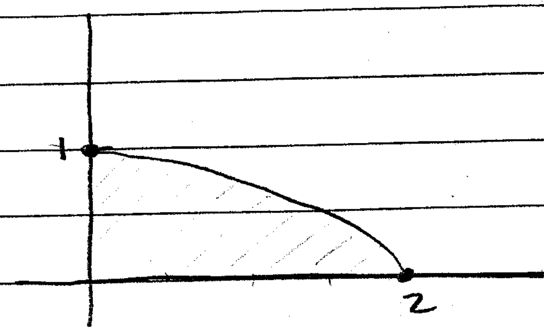
P.3

P283 ex 51 wedge of cheese volume.

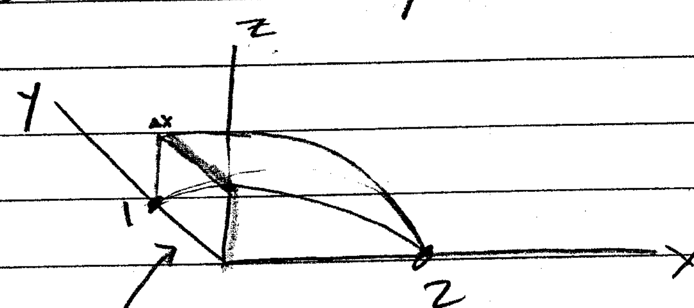


$y = 1 - \frac{x^2}{4}$

cross-sections $\perp$ to x-axis are squares.

The cross sections are in the z direction.

3 dimensionally:



square cross section

Volume of a cross section: $\Delta V = \left(\frac{4-x^2}{4}\right)^2 \Delta x$

So then integrate in x direction

$$V = \int_0^2 \left[\frac{4-x^2}{4}\right]^2 dx$$

$$= \int_0^2 \frac{16 - 8x^2 + x^4}{16}\, dx$$

$$= \frac{1}{16}\int_0^2 16 - 8x^2 + x^4\, dx$$

$$= \frac{1}{16}\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2$$

$\longrightarrow$

$$= \frac{1}{16}\left[\left(32 - \frac{64}{3} + \frac{32}{5}\right) - 0\right]$$

$$= \frac{32}{16}\left[1 - \frac{2}{3} + \frac{1}{5}\right]$$

$$= 2\left(1 - \frac{7}{15}\right)$$

$$= 2\left(\frac{8}{15}\right)$$

$$= \frac{16}{15}$$