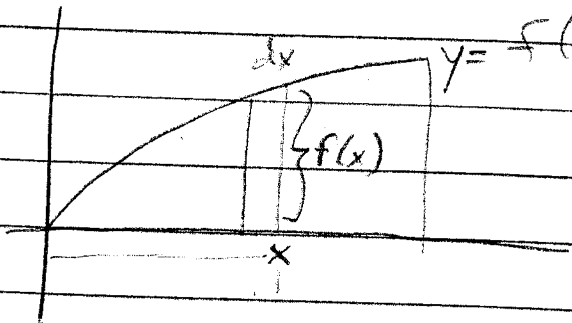
HW [6.2] # 1-15 odds 8/27

What a physicist would do with the HW probs from last night



differential of Area

dA

$dA = f(x)\,dx$

$A = \int dA = \int f(x)\,dx$

$A = \int f(x)\,dx$

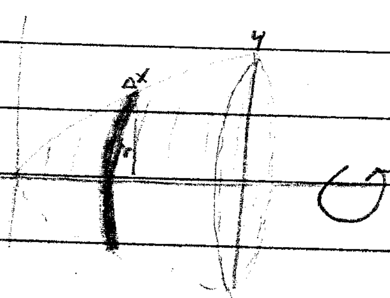
[6.2] volume. Slabs, disks, washers.

Ex: Find Vol of solid formed by rotating plane region R bounded by $y = \sqrt{x}$, the x-axis, the line $x = 4$, when region is rotated about the x-axis.

Soln: $y = \sqrt{x}$

x axis (x)

$x = 4$



we can easily compute vol. of disk.

$V_{disk} = \pi r^2 \cdot (\text{thickness})$

Δx = thickness, radius $= \sqrt{x}$

$\Delta V = \pi [\sqrt{x}]^2 \Delta x.$

Actual Volume $= \int_0^4 \pi [\sqrt{x}]^2 dx$

sum of the infinitely many disks, thickness in limit → zero

$= \pi \int_0^4 x\, dx$

$= \pi \left[\frac{x^2}{2}\right]_0^4$

$= \pi [8 - 0] = \boxed{8\pi}$

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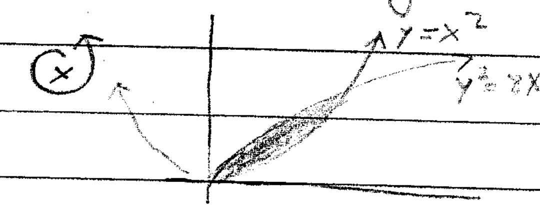
Ex 2

A more complicated region:

$y = x^2$

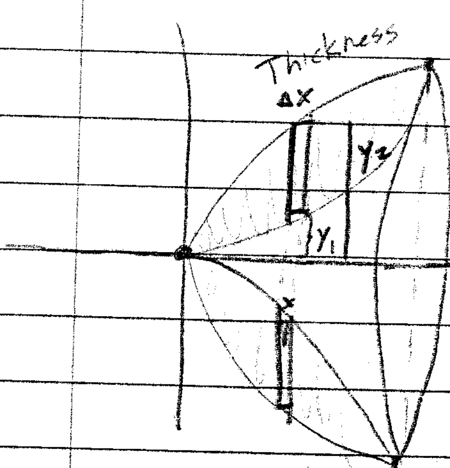
$y^2 = 8x$

Soln:

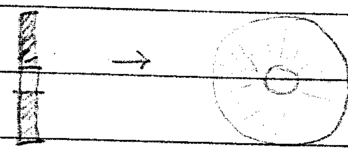


Then
$y^2 = 8x$
$y = \sqrt{8x}$

Rotate it around x axis



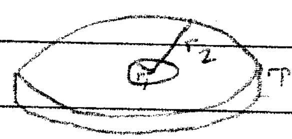
use a disk, Notice that it will have a hole (dotted line).



washer

Make it infinitely thin.
T will be thickness

V_{washer}



$\pi(r_2^2 - r_1^2)T$

$$\Delta V = \pi(y_2^2 - y_1^2)\Delta x$$

$$= \pi\left[(\sqrt{8x})^2 - (x^2)^2\right]\Delta x$$

$$= V = \pi\int_0^2 8x - x^4 \, dx$$

need to find boundaries – pt of intersection

Find
Intersection of
$y = x^2$ and $y^2 = 8x$

so $x^4 = 8x$

$x^4 - 8x = 0$

$x(x^3 - 8) = 0$

Then just integrate it!

$$V = \pi\int_0^2 8x - x^4 \, dx$$

$$V = \pi\left[4x^2 - \frac{x^5}{5}\right]_0^2$$

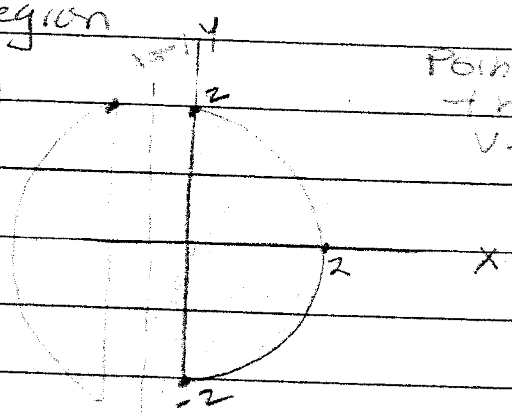
$$V = \pi\left[\left(16 - \frac{32}{5}\right) - (0 - 0)\right]$$

$$V = \frac{48}{5}\pi$$

Ex3 Semi circular region

$x = \sqrt{4-y^2}$ $x=-1$

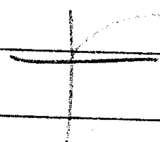
y-axis



Points from trying obvious values.

$x=\sqrt{4-y^2}$ is circle-ish

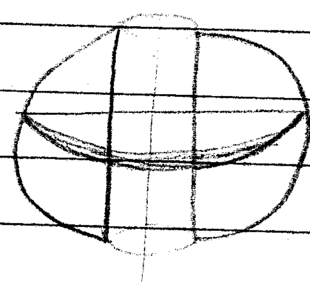
$y=\sqrt{\ }$

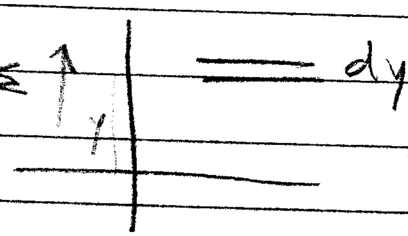


$x=-1$

~~Well that's a sphere with a core taken out~~. No!

Not a sphere!





or