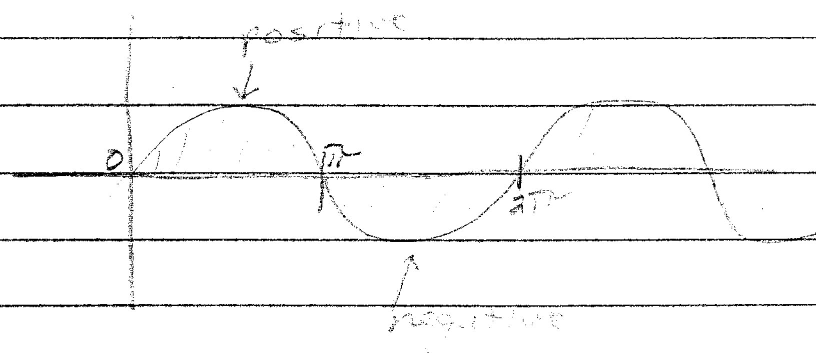
HW [6.1] 1,3,5,9,21 8/26/08

Varberg [6.1], backing up a bit on areas + volumes.

Reminder: Integral has a sign attached



Area of region bounded by $y = \sin x$ and $y = 0$ can't be zero (intuition)

So $A_0^{\pi} + A_{\pi}^{2\pi}$

$$= \int_0^{\pi} \sin x \, dx - \underbrace{\int_{\pi}^{2\pi} \sin x \, dx}_{\text{negative}}$$

So be aware of the graph of whatever the fn is.

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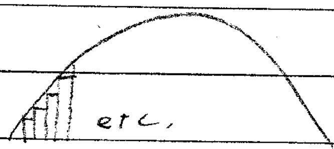
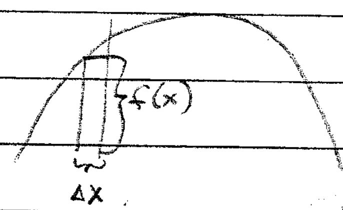
Ex: Get area of region between $y = \sin x$ and $y = 0$
graph on previous page



~~Getting the areas of~~ a bunch of rectangles ~~under the curve will~~ approximate the area ~~under the curve~~



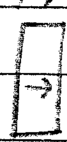
$$\Delta_A = f(x)\Delta x = \sin x \, \Delta x$$

$$A \approx \sum_{i=1}^{n} \sin x \, \Delta x$$

~~well, there is~~ no trouble doing it, just inconvenient. And it's approximate

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \sin x \, dx$$

In the limit the left & right edges of the rectangle coincide.



$$\int_0^{2\pi} \sin x \, dx$$

Underlined parts are operator

$$\int \ldots dx$$

So

$$A = \int_0^{\pi} \sin x \, dx - \int_{\pi}^{2\pi} \sin x \, dx$$

$$= -\cos x \Big]_0^{\pi} - \Big[-\cos x\Big]_{\pi}^{2\pi}$$

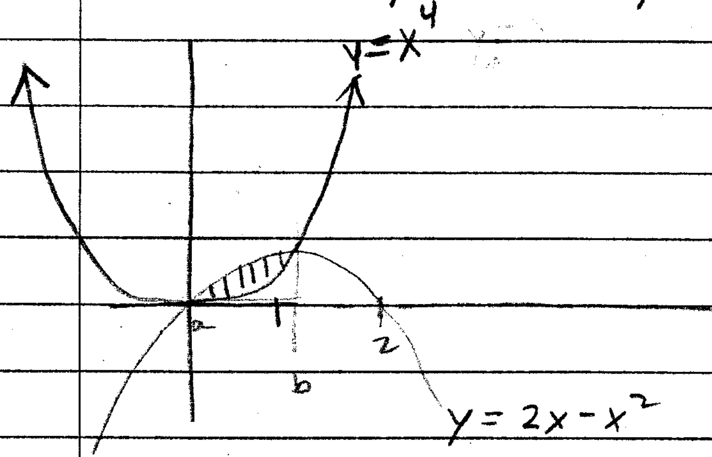
$$= -\cos\pi + \cos 0 - \left[-\cos 2\pi + \cos\pi\right]$$

$$= 0 + \cos 2\pi - \cos\pi$$

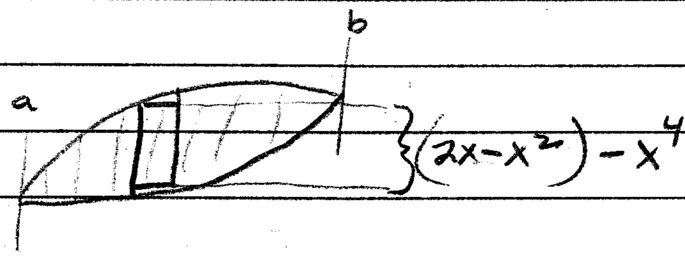
$$= 1 - (-1) = \boxed{2}$$

Another example!

A between $y=x^4$ and $y=2x-x^2=x(2-x)$



zoom in on



$$\Delta_A = \left[(2x-x^2)-x^4\right]\Delta x$$

$$A = \int -x^4-x^2+2x\,dx$$
$$= -\int x^4+x^2-2x\,dx$$

Now a in the diagram is 0 & b is the intersection

$x^4=2x-x^2$ guess the other root:

1 is a good guess.

Now integrat it:

$$-\int_0^1 x^4+x^2-2x\,dx$$

$$= -\left[\tfrac{1}{5}x^5+\tfrac{1}{3}x^3-x^2\right]_0^1 = -\left[\tfrac{1}{5}+\tfrac{1}{3}-1\right] = \boxed{\tfrac{7}{15}}$$

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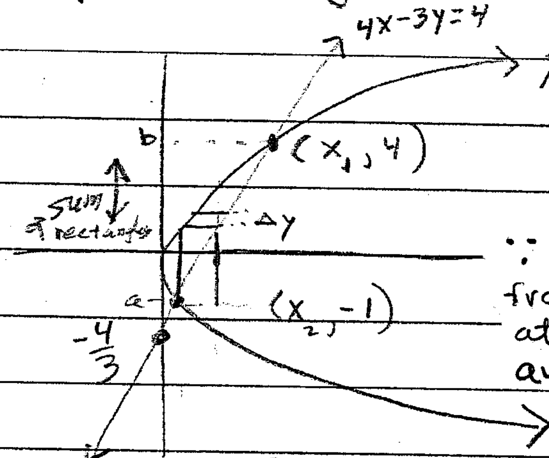
Another example: Find area between $y^2 = 4x$ and the line $4x - 3y = 4$.

Graph is a good thing:



Best to slice it horizontally,

∵ Boundary of region changes from $y^2 = 4x$ to the line $4x-3y=4$ at the point $(\frac{1}{4}, -1)$. At best an inconvenience.

$x = f(y) = \frac{y^2}{4}$ and $x = g(y) = \frac{3y+4}{4}$ write as a fn of y

$$g(y) - f(y) = \left[\frac{3y+4}{4} - \frac{y^2}{4}\right]$$

$$\int \frac{3y+4-y^2}{4}\,dy$$

$4x - 3y = 4$

$y^2 = 4x$

$y^2 - 3y - 4 = 0$

$(y+1)(y-4) = 0$ Intersection when $y = -1$ & $y = 4$