STRATEGY FOR FINDING MAXIMA AND MINIMA

First. When possible, draw a figure to illustrate the problem and label those parts that are important in the problem. Constants and variables should be clearly distinguished.

Second. Write an equation for the quantity that is to be a maximum or a minimum. If this quantity is denoted by y, it is desirable to express it in terms of a single independent variable x. This may require some algebraic manipulation to make use of auxiliary conditions of the problem.

Third. If $y = f(x)$ is the quantity to be a maximum or a minimum, find those values of x for which

$$\frac{dy}{dx} = f'(x) = 0.$$

Fourth. Test each value of x for which $f'(x) = 0$ to determine whether it provides a maximum or minimum or neither. The usual tests are:

a) If $\frac{d^2y}{dx^2}$ is positive when $\frac{dy}{dx} = 0$, y is a minimum.

If $\frac{d^2y}{dx^2}$ is negative when $\frac{dy}{dx} = 0$, y is a maximum.

If $\frac{d^2y}{dx^2} = 0$ when $\frac{dy}{dx} = 0$, the test fails.

b) If

$$\frac{dy}{dx} \text{ is } \begin{cases} \text{positive for } x < x_c, \\ \text{zero for } x = x_c, \\ \text{negative for } x > x_c, \end{cases}$$

then a maximum occurs at x_c. But if dy/dx changes from negative to zero to positive as x advances through x_c, there is a minimum. If dy/dx does not change its sign, neither a maximum nor a minimum need occur.

Fifth. If the derivative fails to exist at some point, examine this point as possible maximum or minimum. (See Fig. 3–17.)

Sixth. If the function $y = f(x)$ is defined for only a limited range of values $a \le x \le b$, examine $x = a$ and $x = b$ for possible extreme values of y. (See Fig. 3–24.)

We find that

$$\frac{dA}{dQ} = -\frac{KM}{Q^2} + \frac{h}{2},$$

which equals zero when

$$Q = \sqrt{\frac{2KM}{h}}.$$

(The negative square root is not a possible value for Q.) The second derivative,

$$\frac{d^2A}{dQ^2} = \frac{2KM}{Q^3},$$

is positive, so that A has a relative minimum at $Q = \sqrt{2KM/h}$. There are no endpoints to worry about because A is defined for all $Q > 0$, and A is differentiable for all $Q > 0$. Hence the minimum is absolute. Incidentally, the graph of A is like the right branch of the graph of $y = x + (1/x)$ in Fig. 3–15.

PROBLEMS

1. Show that the rectangle that has maximum area for a given perimeter is a square.

2. Find the dimensions of the rectangle of greatest area that can be inscribed in a semicircle of radius r.

3. Find the area of the largest rectangle with lower base on the x-axis and upper vertices on the curve $y = 12 - x^2$.

4. An open rectangular box is to be made from a piece of cardboard 8 in. wide and 15 in. long by cutting a square from each corner and bending up the sides. Find the dimensions of the box of largest volume.

5. One side of an open field is bounded by a straight river. How would you put a fence around the other three sides of a rectangular plot in order to enclose as great an area as possible with a given length of fence?

6. An open storage bin with square base and vertical sides is to be constructed from a given amount of material. Determine its dimensions if its volume is a maximum. Neglect the thickness of the material and waste in construction.

7. A box with square base and open top is to hold 32 in^3. Find the dimensions that require the least amount of material. Neglect the thickness of the material and waste in construction.

8. A variable line through the point (1, 2) intersects the x-axis at $A(a, 0)$ and the y-axis at $B(0, b)$. Find the area of the triangle AOB of least area if both a and b are positive.

9. A poster is to contain 50 in^2 of printed matter with margins of 4 in. each at top and bottom and 2 in. at each side. Find the overall dimensions if the total area of the poster is a minimum.

10. A right triangle of given hypotenuse is rotated about one of its legs to generate a right circular cone. Find the cone of greatest volume.

11. It costs a manufacturer c dollars each to manufacture and distribute a certain item. If the items sell at x dollars each, the number sold is given by $n = a/(x - c) + b(100 - x)$, where a and b are certain positive constants. What selling price will bring a maximum profit?

12. A cantilever beam of length L has one end built into a wall, while the other end is simply supported. If the beam weighs w lb per unit length, its deflection y at distance x from the built-in end satisfies the equation

$$48EIy = w(2x^4 - 5Lx^3 + 3L^2x^2),$$

where E and I are constants depending upon the material of the beam and the shape of its cross section. How far from the built-in end does the maximum deflection occur?

13. Determine the constant a in order that the function

$$f(x) = x^2 + \frac{a}{x}$$

may have (a) a relative minimum at $x = 2$, (b) a relative minimum at $x = -3$, (c) a point of inflection at $x = 1$. (d) Show that the function cannot have a relative maximum for any value of a.

14. Determine the constants a and b in order that the function

$$f(x) = x^3 + ax^2 + bx + c$$

may have (a) a relative maximum at $x = -1$ and a relative minimum at $x = 3$, (b) a relative minimum at $x = 4$ and a point of inflection at $x = 1$.

15. A wire of length L is cut into two pieces, one being bent to form a square and the other to form an equilateral triangle. How should the wire be cut (a) if the sum of the two areas is a minimum, (b) if the sum of the areas is a maximum?

16. Find the points on the curve $5x^2 - 6xy + 5y^2 = 4$ that are nearest the origin.

17. Find the point on the curve $y = \sqrt{x}$ nearest the point $(c, 0)$, (a) if $c \geq \frac{1}{2}$, (b) if $c < \frac{1}{2}$.

18. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r.

19. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius r.

20. Show that the volume of the largest right circular cylinder that can be inscribed in a given right circular cone is $\frac{4}{9}$ the volume of the cone.

21. The strength of a rectangular beam is proportional to the product of its width and the square of its depth. Find the dimensions of the strongest beam that can be cut from a circular cylindrical log of radius r.

22. The stiffness of a rectangular beam is proportional to the product of its breadth and the cube of its depth. Find the stiffest beam that can be cut from a log of given diameter.

23. The intensity of illumination at any point is proportional to the product of the strength of the light source and the inverse of the square of the distance from the source. If two sources of relative strengths a and b are a distance c apart, at what point on the line joining them will the intensity be a minimum? Assume the intensity at any point is the sum of intensities from the two sources.

24. A window is in the form of a rectangle surmounted by a semicircle. If the rectangle is of clear glass while the semicircle is of colored glass which transmits only half as much light per square foot as clear glass does, and the total perimeter is fixed, find the proportions of the window that will admit the most light.

25. Right circular cylindrical tin cans are to be manufactured to contain a given volume. There is no waste involved in cutting the tin that goes into the vertical sides of the can, but each end piece is to be cut from a square and the corners of the square wasted. Find the ratio of height to diameter for the most economical cans.

26. A silo is to be made in the form of a cylinder surmounted by a hemisphere. The cost of construction per square foot of surface area is twice as great for the hemisphere as for the cylinder. Determine the dimensions to be used if the volume is fixed and the cost of construction is to be a minimum. Neglect the thickness of the silo and wast in construction.

27. If the sum of the areas of a cube and a sphere constant, what is the ratio of an edge of the cube to th diameter of the sphere when (a) the sum of their volumes a minimum, (b) the sum of their volumes is a maximum?

* **28.** Two towns, located on the same side of a straight rive agree to construct a pumping station and filtering plant the river's edge, to be used jointly to supply the towns wit water. If the distances of the two towns from the river are and b and the distance between them is c, show that the sum of the lengths of the pipe lines joining them to the pumpin station is at least as great as $\sqrt{c^2 + 4ab}$.

* **29.** Light from a source A is reflected to a point B by plane mirror. If the time required for the light to travel fro A to the mirror and then to B is a minimum, show that th angle of incidence is equal to the angle of reflection.

30. Show that a manufacturer's profit is maximized (o minimized) at a level of production where the margin revenue equals the marginal cost.

31. Suppose the government imposes a tax of ten cents, fo each item sold, on the product of Example 7, but oth features are unchanged. To maximize the company's profit how much of the tax should the company absorb and ho much should be passed on to the customer? Why? Com pare the profits before and after the tax.

32. Shipping costs are likely to depend on order size, and might be more realistic in modeling the inventory proble of Example 8 to assume that the setup cost is not a constan Suppose instead that the setup cost is $K + pQ$, the sum of constant and a multiple of Q. What is the most economic quantity to order now?

33. The reaction of the body to a drug is sometimes repr sented by the function

$$R(D) = D^2 \left(\frac{C}{2} - \frac{D}{3}\right),$$

where C = the maximum amount of the drug that could b given, D = the amount given $(0 \leq D \leq C)$, and R = a mea sure of the strength of the reaction (e.g., blood pressur measured in millimeters of mercury). The derivative $R'(D)$ is used as a measure of the sensitivity of the body to th drug. At the point where $R'(D)$ has a maximum, there is th greatest change in R for a small change in D. Show tha this point is $D = C/2$, and that

$$R(C) = 2R\left(\frac{C}{2}\right).$$

[From *Some Mathematical Models in Biology*, p. 221.]

* You may prefer to do these problems without calculus.

41. $x + y = 3$ **42.** $(2x+3)^{-(1/2)}$ **43.** $-6(2-3x), f(x) = (2-3x)^2$ **44.** $-\frac{8}{5}$
45. $6\pi(\Delta r)^2$; a shell around the can with thickness Δr. **46.** $\pi x(20-x)$ **47.** $x_1 = 40, p = 4$ (= 20¢) **49.** $(x-1)(2x-1)$
50. $(2x+1)/9$ **51.** 56 ft/sec **52.** $-\frac{12}{5}$ **53. a)** $h + 2k = 5$ **b)** $h = -4, k = \frac{9}{2}, r = 5\sqrt{5}/2$ **54.** $2\sqrt{x^2+1}$
55. $2/[3(2y+1)(2x+1)\sqrt{x^2+x}]$ **56.** $2x\sqrt{3x^4-1}$ **57.** $(3/(x+1)^2)\sin[((2x-1)/(x+1))^2]$
58. $dy/du = (dy/dx)(dx/du) = 12u\cos(2u^2+2\pi)$. At $u = 0$, $dy/du = 0$. **60.** $3(x-2)/(2x-3)^{3/2})$ **61.** $-\frac{3}{32}$ **62.** $y' = 2$, $y'' = -2$ **64. a)** $3(2x-1)^{-(5/2)}$ **b)** $-162(3x+2)^{-4}$ **c)** $6a$ **66.** $\Delta y = 0.92$, principal part = 0.9
67. a) Length of each side = $2r\sin(\pi/n)$ **b)** $2\pi r$; yes. **68.** $dy/dx = -2x$; $d^2y/dx^2 = -2$ **69.** 14 ± 0.044 ft
70. a) $(x(x+2)/(x+1)^2)\,dx$ **b)** $(x/y)\,dx$ **c)** $(-y/(x+2y))\,dx$
72. $\delta = \epsilon/4$; the function is uniformly continuous for $-2 \le x \le 2$. **75.** $m = \frac{1}{2}, c = \sqrt{2}$

CHAPTER 3

Article 3–1, p. 127

1. Falling $x < \frac{1}{2}$, rising $x > \frac{1}{2}$. Low point at $(\frac{1}{2}, \frac{3}{4})$. **2.** Rising $x < -1$, falling $-1 < x < 2$, rising $x > 2$. High point at $(-1, \frac{3}{2})$, low at $(2, -3)$. **3.** Rising $x < 0$, falling $0 < x < 1$, rising $x > 1$. High at $(0, 3)$, low at $(1, 2)$. **4.** Rising $x < -3$, falling $-3 < x < 3$, rising $x > 3$. High at $(-3, 90)$, low at $(3, -18)$. **5.** Falling $x < -2$, rising $-2 < x < 0$, falling $0 < x < 2$, rising $x > 2$. Low points $(\pm 2, 0)$, high $(0, 16)$. **6.** Defined on $x \neq 0$. Falling $x < 0$. Discontinuity at $x = 0$. Falling $x > 0$. **7.** Rising $-(\pi/2) + 2\pi k < x < (\pi/2) + 2\pi k$. Falling $(\pi/2) + 2\pi k < x < (3\pi/2) + 2\pi k$. High points $((\pi/2) + 2\pi k, 1)$. Low points $((3\pi/2) + 2\pi k, -1)$. (k can be any integer.) **8.** Discontinuous at $x = (\pi/2) + \pi k$. Rising $-(\pi/2) + \pi k < x < (\pi/2) + \pi k$. ($k$ can be any integer.) **9.** Rising all x.

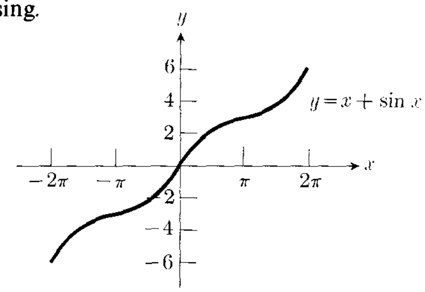
Article 3–2, pp. 131–132

1. $(dA/dt) = 2\pi r(dr/dt)$ **2.** $(dV/dt) = 4\pi r^2(dr/dt)$ **3.** $(8/5\pi)$ ft/min **5.** $(ax+by)/\sqrt{x^2+y^2}$ ft/sec **6.** $(25/9\pi)$ ft/min **7.** $10/\sqrt{21} \approx 2.2$ ft/sec **8.** Increasing 33.7 ft/sec **9.** $(125/144\pi)$ ft/min **10.** $(dy/dt) = -x$. Clockwise. **11.** 8 ft/sec toward the lamp post. Decreasing 3 ft/sec **12.** Increasing 8.75 psi/sec **13.** 1500 ft/sec **14.** 20 ft/sec **15.** Thickness decreasing at the rate of $(5/72\pi)$ in/min. Area decreasing at the rate of $(10/3)$ in^2/min. **16.** $260/\sqrt{37}$ mi/hr.

Article 3–4, pp. 137–138

1. a) $x > 2$ **b)** $x < 2$ **c)** Always **d)** Never $m(2, -1)$ **2. a)** Always **b)** Never **c)** $x < -1$ **d)** $x > -1$ **3. a)** $-1 < x < 1$ **b)** $x < -1, x > 1$ **c)** $x < 0$ **d)** $x > 0$ $M(1, 6); m(-1, 2); I(0, 4)$ **4. a)** $x < -2, x > 3$ **b)** $-2 < x < 3$ **c)** $x > \frac{1}{2}$ **d)** $x < \frac{1}{2}$ $M(-2, \frac{22}{3}); m(3, -\frac{27}{2}); I(\frac{1}{2}, -\frac{37}{12})$ **5. a)** $x < -2, x > 2$ **b)** $-2 < x < 2, x \neq 0$ **c)** $x > 0$ **d)** $x < 0$. $M(-2, -4); m(2, 4)$ **6. a)** $-\pi + 2\pi k < x < 2\pi k$ **b)** $2\pi k < x < \pi + 2\pi k$ **c)** $(\pi/2) + 2\pi k < x < (3\pi/2) + 2\pi k$ **d)** $-(\pi/2) + 2\pi k < x < (\pi/2) + 2\pi k$; $M(2\pi k, 1)$; $m(\pi + 2\pi k, -1)$; $I((\pi/2) + \pi k, 0)$; k any integer **9.** $M(-1, 7)$ **10.** $m(1, 1)$
11. $M(-2, 28); m(2, -4); I(0, 12)$ **12.** $M(0, 2); m(2, -2); I(1, 0)$
13. $M(0, 0)$ **14.** $m(2, 0)$ **15.** $(1, -1)$ **16.** $(16, -3), (-16, 1)$
17. $(2, 0), (1, \pm 1)$ **18.** $(3, 1)$ **21.** Concave downward because $f''(x) = -(1/x^2)$ is negative. **23. a)** 3 times; near $x = -1.2, -0.4, +0.8$ **b)** Once; near $x = -1.8$ **c)** Once; near $x = 1.2$

24. $dy/dx = 1 + \cos x = 0$ at $x = \pi + 2\pi k$; elsewhere the slope is positive and the graph is rising.



Article 3–6, pp. 151–153

2. $(r\sqrt{2}) \times (r/\sqrt{2})$ **3.** 32 **4.** $\frac{5}{3} \times \frac{14}{3} \times \frac{35}{3}$ (inches) **5.** Use one-half of the fence parallel to the river.
6. $\sqrt{A/3} \times \sqrt{A/3} \times \frac{1}{2}\sqrt{A/3}$, where A = given amount of material. **7.** 4″ × 4″ × 2″ **8.** 4 **9.** 9 inches wide and 18 inches high
10. $V_{max} = 2\pi h^3/(9\sqrt{3})$, h = given hypotenuse **11.** $(100 + c)/2$ **12.** $0.58L$
13. a) 16 **b)** −54 **c)** −1 **d)** $f'(x) = 0$ when $2x^3 = a$ and then $f''(x) = +6$ or $+2$, according as $a \neq 0$ or $a = 0$.

14. a) $a = -3, b = -9$ **b)** $a = -3, b = -24$ **15. a)** $4L/(4 + 3\sqrt{3})$ for the square, $3\sqrt{3}\,L/(4 + 3\sqrt{3})$ for the triangle **b)** Use it all for the square **16.** $(\frac{1}{2}, -\frac{1}{2})$ and $(-\frac{1}{2}, \frac{1}{2})$ **17. a)** $(c - \frac{1}{2}, \sqrt{c - \frac{1}{2}})$ **b)** $(0, 0)$ **18.** $\frac{32}{81}\pi r^3$ **19.** $\frac{4}{9}\pi r^3\sqrt{3}$
21. Width $= 2r/\sqrt{3}$, depth $= 2r\sqrt{\frac{2}{3}}$ **22.** Width of beam = radius of log. **23.** $x/(c - x) = \sqrt[3]{a/b}$, where x is the distance from the source of strength a. **24.** Height of rectangle $= (4 + \pi)/8$ times diameter of semicircle **25.** $4/\pi$ **26.** Diameter = altitude of cylinder $= \sqrt[3]{3V/\pi}$, V = total volume **27. a)** Unity **b)** Zero **31.** Before tax: Production = 7,500; price = $1.25; profit = $5,425. After tax: Production = 7,000; price = $1.30; profit = $4,700. To maximize profit the company should absorb 5¢ of the tax and add 5¢ to the price. **32.** Still $Q = \sqrt{2KM/h}$. Of course the average weekly cost increases if $p > 0$. **35.** Maxima: $x = (\pi/4) + 2\pi k$, k any integer, $y = \sqrt{2}$. Minima: $x = (5\pi/4) + 2\pi k$, k any integer, $y = -\sqrt{2}$. [*Note.* $\sin x + \cos x = \sqrt{2}\sin[x + (\pi/4)]$.]

Article 3–7, p. 155

6. The function is not continuous on the closed interval $0 \le x \le 1$.

Article 3–8, pp. 159–160

1. $\frac{1}{2}$ **2.** $\sqrt{3}$ **3.** $\frac{8}{27}$ **4.** 1 **5.** $\frac{3}{2}$ **7. a)** $3\frac{1}{6}$ **b)** 4.012 **c)** 0.0101

Article 3–9, p. 165

1. $\frac{1}{4}$ **2.** 2 **3.** $\frac{5}{7}$ **4.** $\frac{3}{11}$ **5.** 0 **6.** -2 **7.** 5 **8.** $-\frac{1}{2}$ **9.** 1 **10.** $\frac{1}{4}$ **11.** $\frac{1}{6}$ **12.** $\frac{1}{2}$ **13.** $+\infty$ **14.** 3 **15.** $\cos a$ **16.** an **17.** $+\infty$ **18.** $-\frac{1}{2}$ **19.** (b) is correct. L'Hôpital's rule does not apply because the limit of the denominator is finite and nonzero. **20.** -1 **21.** $(a + b)/2$ **22.** $(-1 + \sqrt{37})/3 \approx 1.694$

Miscellaneous Problems Chapter 3, pp. 171–174

1. a) $x < \frac{9}{2}$ **b)** $x > \frac{9}{2}$ **c)** No values **d)** All values **2. a)** $x > 3, x < \frac{1}{3}$ **b)** $\frac{1}{3} < x < 3$ **c)** $x > \frac{5}{3}$ **d)** $x < \frac{5}{3}$
3. a) $x < 3$ **b)** $x > 3$ **c)** $0 < x < 2$ **d)** $x < 0, x > 2$ **4. a)** $|x| > \frac{1}{2}$ **b)** $|x| < \frac{1}{2}$ **c)** $x > 0$ **d)** $x < 0$
5. a) $x > \sqrt[3]{2}$ **b)** $x < \sqrt[3]{2}$ **c)** $x > 0, x < -\sqrt[3]{4}$ **d)** $-\sqrt[3]{4} < x < 0$ **6. a)** $x < 0, x > 2$ **b)** $0 < x < 2$ **c)** $x \neq 0$ **d)** Never
7. a) $x < 0$ **b)** $x > 0$ **c)** $x \neq 0$ **d)** Never **8. a)** $x \neq -1$ **b)** Never **c)** $x < -1$ **d)** $x > -1$ **9. a)** $x \neq 0$ **b)** Never **c)** $x < 0$ **d)** $x > 0$ **10. a)** $-1 < x < 0, x > 1$ **b)** $x < -1, 0 < x < 1$ **c)** $x < -1/\sqrt{3}, x > 1/\sqrt{3}$ **d)** $|x| < 1/\sqrt{3}$
11. a) $x < -2b/a, x > 0$ **b)** $-2b/a < x < 0$ **c)** $x > -b/a$ **d)** $x < -b/a$
12. a) $x < 1, x > 2$ **b)** $1 < x < 2$ **c)** $x > \frac{3}{2}$ **d)** $x < \frac{3}{2}$ **13. a)** $x < -1, x > \frac{1}{3}$ **b)** $-1 < x < \frac{1}{3}$ **c)** $x > -\frac{1}{3}$ **d)** $x < -\frac{1}{3}$
14. a) $0 < x < 4$ **b)** $x < 0, x > 4$ **c)** $x < 2$ **d)** $x > 2$
15. a) At $x = 1$; because y' goes from + to −. **b)** At $x = 3$; because y' goes from − to +.
17. If $v = k/\sqrt{s}$, then $dv/dt = -k^2/2s^2$ **18.** $k^2/2$ **19.** $\frac{1}{4}$ in/min **20.** $(3/400\pi)$ ft/min
21. $dr/dt = -(3/400)$ ft/min. $dA/dt = -\frac{6}{5}\pi$ ft²/min. $\Delta r \approx -(3/4000)$ ft; $\Delta A \approx -(3\pi/25)$ ft²
22. a) Approx. 606 mi/hr **b)** Approx. 0.83 mi **23.** $\frac{1}{2}\sqrt{3}$ **24.** $-480\sqrt{2}/7$ **25.** Yes, it will fill, because $dy/dt > 0.007$ for $y \le 10$.
26. $a = -gR^2/s^2$ **28.** $(\sqrt{3}/30)r$, increasing **29.** ≈ 18 mi/hr **30.** $(8/9\pi)$ ft/min **31.** $\frac{20}{3}$ and $\frac{40}{3}$ **32.** 10 **33.** 18 and 18. No.
34. $a = 1; b = -3; c = -9; d = 5$ **35.** $\frac{1}{2}$ **36.** $r = 25$ ft **37.** $t = 1, s = 16$ **38.** $r = 4, h = 4$ **39.** Diameter = height = 4 in
40. $4\sqrt{3}$ **41.** 276 (approx.) **42. a)** $(2, \pm\sqrt{3})$ **b)** and **c)** $(1, 0)$ **43. a)** $2\frac{1}{12}$ mi from A; **b)** $2\frac{1}{12}$ mi from A; **c)** at B
44. a) True **b)** False **c)** True **d)** False **45.** Each $= (P - b)/2$ **46.** Each $= \tan^{-1}(4k/b^2)$ **48.** $m = \frac{1}{4}$
50. Sides 100 m; radius of semicircular ends $100/\pi$ m **57.** 6 × 18 ft **58.** Approx. 16.4 in. **59.** $(h^{2/3} + w^{2/3})^{3/2}$ m
60. $v = \sqrt[n]{a/((n - 1)b)}$ **61.** $r = \sqrt{A}, \theta = 2$ rad **62.** Approx. 1.94 gal **63. a)** A decreasing 0.04π cm²/sec **b)** $t = (5b - 3a)/(a^2 - b^2)$ **64.** $t = (R - cr)/(ca - b)$, where $c = \sqrt{a/b}$ **66. a)** True **b)** Not necessarily true: $f(x) = g(x) = 5 + x + x^3$ for $|x| \le 1, a = 0$. **67.** $x = (1/n)\sum_{i=1}^{n} c_i$ **68.** $m = \frac{5}{7}$ **70.** Doesn't apply; derivative doesn't exist at $x = 0$. **72.** $c = \pm((b^{2/3} + a^{1/3}b^{1/3} + a^{2/3})/3)^{3/2}$ **73.** Root -0.67 **75. a)** $x = (c - b)/(2e)$ **b)** $P = \frac{1}{2}(b + c)$ **c)** $-a + (c - b)^2/(4e)$ **d)** $P = \frac{1}{2}(b + t + c)$; that is, it adds $\frac{1}{2}t$ to its previous price. **76. a)** $\frac{10}{3}$ **b)** $\frac{5}{3}$ **c)** $\frac{1}{2}$ **d)** 0

CHAPTER 4

Article 4–2, p. 181

1. $y = (x^3/3) + x + C$ **2.** $y = -(1/x) + (x^2/2) + C$ **3.** $y^2 = x^2 + C$ **4.** $3y^{1/2} = x^{3/2} + C$ **5.** $y^{2/3} = x^{2/3} + C$ **6.** $-(1/y) = x^2 + C$ **7.** $y = x^3 - x^2 + 5x + C$ **8.** $s = t^3 + 2t^2 - 6t + C$ **9.** $r = (2z + 1)^4/8 + C$ **10.** $-u^{-1} = 2v^4 - 4v^{-2} + C$
11. $x^{1/2} = 4t + C$ **12.** $y = 4t^3/3 + 4t - t^{-1} + C$ **13.** $y = z^3/3 - z^{-1} + C$ **14.** $x^2 + 3x + C$ **15.** $(x^3/3) - (2x^{3/2}/3) + C$