

[08-02-18-T-Phy]
Vectors - Review 2 & Energy

The point O is to be taken as the origin $O(0, 0)$.

■ **A. Write the vector equation for the line through the points given.**

[1] $A(2, 3), B(7, 13)$

[2] $A(-5, 3), B(2, -9)$

[3] $P(-17, -30), Q(-3, -9)$

[4] $P(12, -13), Q(3, -16)$

[5] $R(5, 7), S(-8, -15)$

■ **B. Answer the following**

[1] If ABCD is a parallelogram where $A(3, 7), B(5, 7), C(5, 2)$. Find the coordinates of the point D.

[2] Points A, B, and C are collinear and the distance from A to C is 5 times the distance from A to B. If the coordinates of A and B are $A(-2, 5), B(7, -11)$, find the coordinates of point C.

[3] Points A, B, and C are collinear and the distance from A to C is $\frac{1}{10}$ the distance from A to B. If the coordinates of A and B are $A(-2, 5), B(7, -11)$, find the coordinates of point C.

[4] Find the unit vector of $\vec{v} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

[5] Find the unit vector of $\vec{v} = \begin{bmatrix} -5 \\ 12 \end{bmatrix}$

■ **C. Answer the following. Assume that $g = 10 \frac{m}{s^2}$.**

[1] A 2 kg block is carried to the top of a ramp of height 12m. What is the work done in lifting the block?

[2] A 2 kg block is carried to the top of a spiral staircase of height 15m. What is the potential energy of the block?

[3] A 6 kg block is carried to the top of a spiral staircase of height 10m. It is then dropped. What is the velocity of the block when it hits the floor at the bottom of the stairs?

A. [1] $\vec{r} = \langle 2, 3 \rangle + t \langle 5, 10 \rangle$
 $= \langle 2, 3 \rangle + t \langle 1, 2 \rangle$

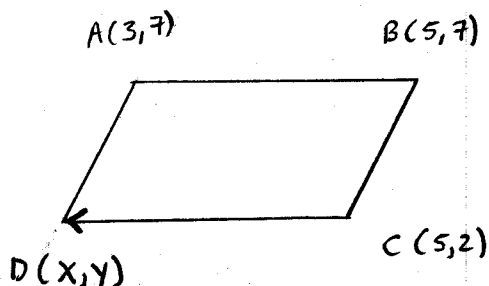
[2] $\vec{r} = \langle -5, 3 \rangle + t \langle 7, -12 \rangle$

[3] $\vec{r} = \langle -17, -30 \rangle + t \langle 14, 21 \rangle$

[4] $\vec{r} = \langle 12, -13 \rangle + t \langle -9, -3 \rangle$

[5] $\vec{r} = \langle 5, 7 \rangle + t \langle -13, -22 \rangle$

B [1]



SOLN (one of many)

$$\vec{OD} = \vec{OC} + \vec{CD}$$

$$\vec{CD} = \vec{BA}$$

\therefore ABCD parallelogram (opposite sides equal and parallel)

$$\vec{OD} = \vec{OC} + \vec{BA}$$

$$= \begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

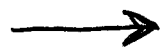
$$\vec{OD} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\therefore D(3, 2)$$

[2] $\vec{OC} = \vec{OA} + 5 \vec{AB}$

$$= \langle -2, 5 \rangle + 5 \langle 9, -16 \rangle$$

$$= \langle 43, -75 \rangle$$



$$\begin{aligned}
 [3] \quad \vec{OC} &= \vec{OA} + \frac{1}{10} \vec{AB} \\
 &= \langle -2, 5 \rangle + \frac{1}{10} \langle 9, -16 \rangle \\
 &= \langle -1.1, 3.4 \rangle
 \end{aligned}$$

$$[4] \quad \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3, 7 \rangle}{\sqrt{9+49}} = \left\langle \frac{3}{\sqrt{58}}, \frac{7}{\sqrt{58}} \right\rangle$$

$$[5] \quad \hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle -5, 12 \rangle}{\sqrt{169}} = \left\langle \frac{-5}{13}, \frac{12}{13} \right\rangle$$

$$C. [1] \quad W = \vec{F} \cdot \vec{d} = 2(\text{kg}) \left(10 \frac{\text{m}}{\text{s}^2}\right) (12 \text{ m}) = 240 \text{ Nm} = 240 \text{ J}$$

$$[2] \quad U = mgh = (2 \text{ kg}) (10 \text{ m s}^{-2}) (15 \text{ m}) = 300 \text{ J}$$

$$[3] \quad U \text{ at top} = mgh = (6 \text{ kg}) (10 \text{ m s}^{-2}) (10 \text{ m}) = 600 \text{ J}$$

Dropped. At bottom ($h=0$) all potential energy converted to kinetic energy.

So,

$$mgh = \frac{1}{2} m v^2$$

$$\Rightarrow v^2 = 2gh \frac{\text{m}}{\text{s}^2} \cdot \text{m}$$

$$= \sqrt{2 \left(10 \frac{\text{m}}{\text{s}^2}\right) (10 \text{ m})}$$

$$= \sqrt{200} \frac{\text{m}}{\text{s}}$$

$$\therefore v = 10\sqrt{2} \text{ m/s}$$

$$\approx 14.14 \frac{\text{m}}{\text{s}}$$