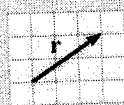
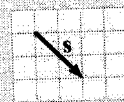
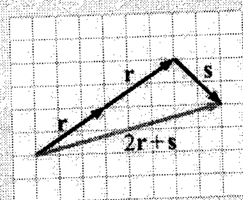
Example 10

Given vectors 

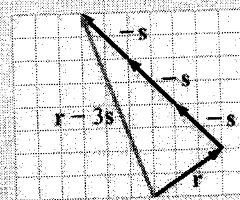
and 

show how to find **a** $2\mathbf{r}+\mathbf{s}$ **b** $\mathbf{r}-3\mathbf{s}$ geometrically.

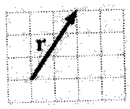
a

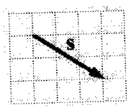


b



EXERCISE 15B.3

1 Given vectors 

and 

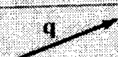
, show how to find geometrically:

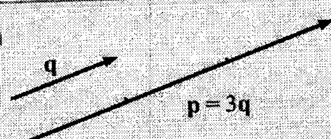
a $-\mathbf{r}$ **b** $2\mathbf{s}$ **c** $\frac{1}{2}\mathbf{r}$ **d** $-\frac{3}{2}\mathbf{s}$

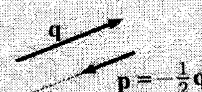
e $2\mathbf{r}-\mathbf{s}$ **f** $2\mathbf{r}+3\mathbf{s}$ **g** $\frac{1}{2}\mathbf{r}+2\mathbf{s}$ **h** $\frac{1}{2}(\mathbf{r}+3\mathbf{s})$

Example 11

Draw sketches of vectors **p** and **q** if **a** $\mathbf{p}=3\mathbf{q}$ **b** $\mathbf{p}=-\frac{1}{2}\mathbf{q}$.

Let **q** be 

a 

b 

2 Draw sketches of **p** and **q** if:

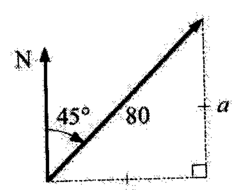
a $\mathbf{p}=\mathbf{q}$ **b** $\mathbf{p}=-\mathbf{q}$ **c** $\mathbf{p}=2\mathbf{q}$ **d** $\mathbf{p}=\frac{1}{3}\mathbf{q}$ **e** $\mathbf{p}=-3\mathbf{q}$

C 2-D VECTORS IN COMPONENT FORM

So far we have examined vectors from their geometric representation.

We have used arrows where:

- the **length** of the arrow represents size (magnitude)
- the **arrowhead** indicates direction.

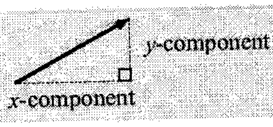


$$a^2+a^2=80^2$$
$$\therefore\ 2a^2=6400$$
$$\therefore\ a^2=3200$$
$$\therefore\ a \doteqdot 56.6$$

Consider a car travelling at 80 km/h in a NE direction.

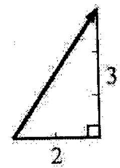
The velocity vector could be represented by using the x and y-steps which are necessary to go from the start to the finish.

In this case the column vector $\begin{bmatrix} 56.6 \\ 56.6 \end{bmatrix}$ gives the x and y steps.



$\begin{bmatrix} x \\ y \end{bmatrix}$ is the **component form** of a vector.

For example, given $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ we could draw



and vice versa.

2 is the horizontal step and
3 is the vertical step.

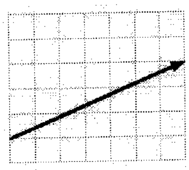
EXERCISE 15C.1

1 Draw arrow diagrams to represent the vectors:

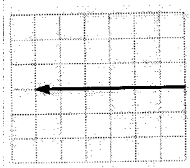
a $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ **b** $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ **d** $\begin{bmatrix} -1 \\ -3 \end{bmatrix}$

2 Write the illustrated vectors in component form:

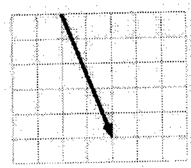
a



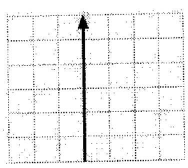
b



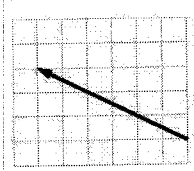
c



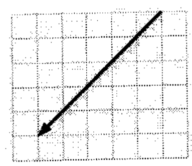
d



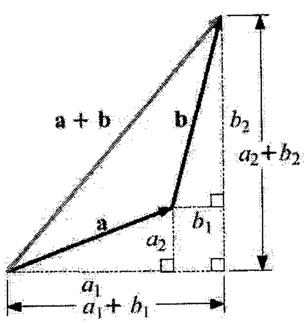
e



f



VECTOR ADDITION



Consider adding vectors $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

Notice that the

horizontal step for $\mathbf{a} + \mathbf{b}$ is $a_1 + b_1$ and the

vertical step for $\mathbf{a} + \mathbf{b}$ is $a_2 + b_2$.

So, if $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ then $\mathbf{a} + \mathbf{b} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$.

Example 12

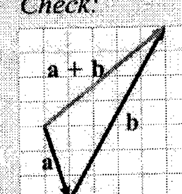
If $\mathbf{a} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

find $\mathbf{a} + \mathbf{b}$.

Check graphically.

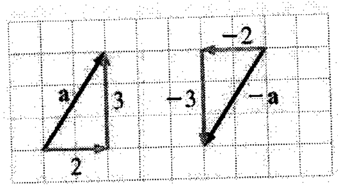
$$\mathbf{a} + \mathbf{b} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$
$$= \begin{bmatrix} 1+4 \\ -3+7 \end{bmatrix}$$
$$= \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

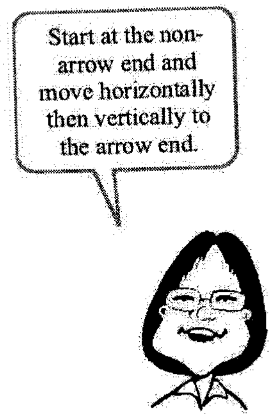
Check:



NEGATIVE VECTORS

Consider the vector $\mathbf{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.





Notice that $-\mathbf{a} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$.

In general, if $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ then $-\mathbf{a} = \begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix}$.

ZERO VECTOR

The zero vector is $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and for any vector $\mathbf{a}$, $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$.

VECTOR SUBTRACTION

To subtract one vector from another, we simply **add its negative**, i.e., $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$.

Notice that, if $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ then

$$\begin{aligned} \mathbf{a} - \mathbf{b} &= \mathbf{a} + (-\mathbf{b}) \\ &= \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} -b_1 \\ -b_2 \end{bmatrix} \\ &= \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \end{bmatrix} \end{aligned}$$

i.e., if $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ then $\mathbf{a} - \mathbf{b} = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \end{bmatrix}$.

EXERCISE 15C.2

1 If $\mathbf{a} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$ find:

a $\mathbf{a} + \mathbf{b}$ **b** $\mathbf{b} + \mathbf{a}$ **c** $\mathbf{b} + \mathbf{c}$ **d** $\mathbf{c} + \mathbf{b}$
e $\mathbf{a} + \mathbf{c}$ **f** $\mathbf{c} + \mathbf{a}$ **g** $\mathbf{a} + \mathbf{a}$ **h** $\mathbf{b} + \mathbf{a} + \mathbf{c}$

Example 13

Given $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$ find:

a $\mathbf{q} - \mathbf{p}$
b $\mathbf{p} - \mathbf{q} - \mathbf{r}$

a $\mathbf{q} - \mathbf{p}$

$$= \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 1-3 \\ 4+2 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

b $\mathbf{p} - \mathbf{q} - \mathbf{r}$

$$= \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 3-1--2 \\ -2-4--5 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

2 Given $\mathbf{p} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ find:

a $\mathbf{p} - \mathbf{q}$
b $\mathbf{q} - \mathbf{r}$
c $\mathbf{p} + \mathbf{q} - \mathbf{r}$
d $\mathbf{p} - \mathbf{q} - \mathbf{r}$
e $\mathbf{q} - \mathbf{r} - \mathbf{p}$
f $\mathbf{r} + \mathbf{q} - \mathbf{p}$

3 a Given $\overrightarrow{BA} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\overrightarrow{BC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ find $\overrightarrow{AC}$. **Hint:** $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{BA} + \overrightarrow{BC}$.

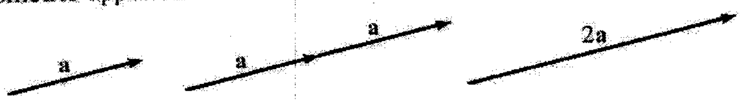
b If $\overrightarrow{AB} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\overrightarrow{CA} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, find $\overrightarrow{CB}$.

c If $\overrightarrow{PQ} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, $\overrightarrow{RQ} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\overrightarrow{RS} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, find $\overrightarrow{SP}$.

SCALAR MULTIPLICATION

Recall the geometric approach for scalar multiplication.

For example:



A **scalar** is a non-vector quantity.

The word scalar is also used for a constant number.

Consider $\mathbf{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

$$\mathbf{a} + \mathbf{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \text{ and}$$

$$\mathbf{a} + \mathbf{a} + \mathbf{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

Examples like these suggest the following definition for **scalar multiplication**:

If k is a scalar, then $k\mathbf{a} = \begin{bmatrix} ka_1 \\ ka_2 \end{bmatrix}$.

Notice that:

- $(-1)\mathbf{a} = \begin{bmatrix} (-1)a_1 \\ (-1)a_2 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix} = -\mathbf{a}$
- $(0)\mathbf{a} = \begin{bmatrix} (0)a_1 \\ (0)a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathbf{0}$

Example 14

For $\mathbf{p} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ find: a $3\mathbf{q}$ b $\mathbf{p} + 2\mathbf{q}$ c $\frac{1}{2}\mathbf{p} - 3\mathbf{q}$

a $3\mathbf{q}$

$$= 3\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 6 \\ -9 \end{bmatrix}$$

b $\mathbf{p} + 2\mathbf{q}$

$$= \begin{bmatrix} 4 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 + 2(2) \\ 1 + 2(-3) \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

c $\frac{1}{2}\mathbf{p} - 3\mathbf{q}$

$$= \frac{1}{2}\begin{bmatrix} 4 \\ 1 \end{bmatrix} - 3\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(4) - 3(2) \\ \frac{1}{2}(1) - 3(-3) \end{bmatrix} = \begin{bmatrix} -4 \\ 9\frac{1}{2} \end{bmatrix}$$

EXERCISE 15C.3

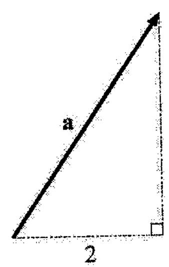
1 For $\mathbf{p} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$ find:

a $-3\mathbf{p}$ **b** $\frac{1}{2}\mathbf{q}$ **c** $2\mathbf{p} + \mathbf{q}$ **d** $\mathbf{p} - 2\mathbf{q}$
e $\mathbf{p} - \frac{1}{2}\mathbf{r}$ **f** $2\mathbf{p} + 3\mathbf{r}$ **g** $2\mathbf{q} - 3\mathbf{r}$ **h** $2\mathbf{p} - \mathbf{q} + \frac{1}{3}\mathbf{r}$

2 If $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ find by diagram (and comment on the results):

a $\mathbf{p} + \mathbf{p} + \mathbf{q} + \mathbf{q} + \mathbf{q}$ **b** $\mathbf{p} + \mathbf{q} + \mathbf{p} + \mathbf{q} + \mathbf{q}$ **c** $\mathbf{q} + \mathbf{p} + \mathbf{q} + \mathbf{p} + \mathbf{q}$

LENGTH OF A VECTOR



Consider vector $\mathbf{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ as illustrated.

Recall that $|\mathbf{a}|$ represents the length of **a**.

By Pythagoras $|\mathbf{a}|^2 = 2^2 + 3^2 = 4 + 9 = 13$

$\therefore \quad |\mathbf{a}| = \sqrt{13}$ units

In general, if $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, then $|\mathbf{a}| = \sqrt{{a_1}^2 + {a_2}^2}$.

Example 15

If $\mathbf{p} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ find: **a** $|\mathbf{p}|$ **b** $|\mathbf{q}|$ **c** $|\mathbf{p} - 2\mathbf{q}|$

a $\mathbf{p} = \begin{bmatrix} 3 \\ -5 \end{bmatrix} \therefore |\mathbf{p}| = \sqrt{9+25}$
$= \sqrt{34}$ units

b $\mathbf{q} = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \therefore |\mathbf{q}| = \sqrt{1+4}$
$= \sqrt{5}$ units

c $\mathbf{p} - 2\mathbf{q} = \begin{bmatrix} 3 \\ -5 \end{bmatrix} - 2\begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ $\therefore |\mathbf{p} - 2\mathbf{q}| = \sqrt{5^2 + (-1)^2}$
$= \sqrt{26}$ units

EXERCISE 15C.4

1 For $\mathbf{r} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ find:

a $|\mathbf{r}|$ **b** $|\mathbf{s}|$ **c** $|\mathbf{r} + \mathbf{s}|$ **d** $|\mathbf{r} - \mathbf{s}|$ **e** $|\mathbf{s} - 2\mathbf{r}|$

2 If $\mathbf{p} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ find:

a $|\mathbf{p}|$ **b** $|2\mathbf{p}|$ **c** $|-2\mathbf{p}|$ **d** $|3\mathbf{p}|$ **e** $|-3\mathbf{p}|$
f $|\mathbf{q}|$ **g** $|4\mathbf{q}|$ **h** $|-4\mathbf{q}|$ **i** $|\frac{1}{2}\mathbf{q}|$ **j** $|-\frac{1}{2}\mathbf{q}|$

3 From your answers in **2**, you should have noticed that $|k\mathbf{a}| = |k|\,|\mathbf{a}|$
i.e., (the length of $k\mathbf{a}$) = (the modulus of k) × (the length of **a**).

By letting $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, prove that $|k\mathbf{a}| = |k|\,|\mathbf{a}|$.

ANS TO
[07-11-12: HH-PHY]

b Conjecture is: $A^n = \begin{bmatrix}1 & 3^n-1\\0 & 3^n\end{bmatrix}$, $n \in Z^+$

d Yes as $A^{-1} = \begin{bmatrix}1 & -\frac{2}{3}\\0 & \frac{1}{3}\end{bmatrix}$.

3 **a** $P^2 = \begin{bmatrix}3 & 2\\-2 & -1\end{bmatrix}$, $P^3 = \begin{bmatrix}4 & 3\\-3 & -2\end{bmatrix}$, $P^4 = \begin{bmatrix}5 & 4\\-4 & -3\end{bmatrix}$

b **i** $P^n = \begin{bmatrix}n+1 & n\\-n & 1-n\end{bmatrix}$ for all $n \in Z^+$

REVIEW SET 14A

1 **a** $\begin{bmatrix}4 & 2\\-2 & 3\end{bmatrix}$ **b** $\begin{bmatrix}9 & 6\\0 & -3\end{bmatrix}$ **c** $\begin{bmatrix}-2 & 0\\4 & -8\end{bmatrix}$ **d** $\begin{bmatrix}2 & 2\\2 & -5\end{bmatrix}$

e $\begin{bmatrix}-5 & -4\\-2 & 6\end{bmatrix}$ **f** $\begin{bmatrix}7 & 6\\4 & -11\end{bmatrix}$ **g** $\begin{bmatrix}-1 & 8\\2 & -4\end{bmatrix}$ **h** $\begin{bmatrix}3 & 2\\-6 & -8\end{bmatrix}$

i $\begin{bmatrix}\frac{1}{3} & \frac{2}{3}\\0 & -1\end{bmatrix}$ **j** $\begin{bmatrix}9 & 4\\0 & 1\end{bmatrix}$ **k** $\begin{bmatrix}-3 & -10\\6 & 8\end{bmatrix}$ **l** $\begin{bmatrix}\frac{1}{3} & \frac{2}{3}\\\frac{1}{6} & \frac{1}{12}\end{bmatrix}$

2 **a** $a = 0, b = 5, c = 1, d = -4$
b $a = 2, b = -1, c = 3, d = 8$

3 **a** $Y = B - A$ **b** $Y = \frac{1}{2}(D - C)$ **c** $Y = A^{-1}B$
d $Y = CB^{-1}$ **e** $Y = A^{-1}(C - B)$ **f** $Y = B^{-1}A$

4 **a** $x = 0, y = -\frac{1}{2}$ **b** $x = \frac{12}{7}, y = \frac{13}{7}$ **c** $X = \begin{bmatrix}-1 & 8\\-2 & 6\end{bmatrix}$

d $X = \begin{bmatrix}-\frac{1}{2}\\\frac{3}{2}\end{bmatrix}$ **e** $X = \begin{bmatrix}\frac{14}{3}\\\frac{1}{3}\end{bmatrix}$ **f** $X = \begin{bmatrix}\frac{1}{2} & \frac{3}{2}\\\frac{3}{2} & -\frac{1}{2}\end{bmatrix}$

5 **a** $\begin{bmatrix}4 & 8\\0 & 2\\6 & 4\end{bmatrix}$ **b** $\begin{bmatrix}1 & 2\\0 & \frac{1}{2}\\\frac{3}{2} & 1\end{bmatrix}$ **c** $[\,11 \quad 12\,]$ **d** BA does not exist.

6 **a** $\begin{bmatrix}4 & 2\\2 & 4\\3 & 4\end{bmatrix}$ **b** $\begin{bmatrix}2 & -2\\0 & 4\\-1 & -2\end{bmatrix}$ **c** $\begin{bmatrix}-\frac{3}{2} & 3\\\frac{1}{2} & -4\\2 & \frac{7}{2}\end{bmatrix}$

7 unique solution if $k \neq \frac{3}{4}$, no solution if $k = \frac{3}{4}$

8 $x = 1, \; y = -2, \; z = -1$

9 **a** $-2a + 4b + c = -20$
$a + 3b + c = -10$
$\therefore \; a = 2 - t, \; b = -4 - 3t, \; c = 10t$ (t is real)

b There are three unknowns and only two pieces of information. **c** $x^2 + y^2 + 4x + 2y - 20 = 0$

10 When $k \neq 27$, there are no solutions.
When $k = 27$, there are infinite solutions of the form $x = 2 - t, \; y = 2t + 3, \; z = t$ (t is real).

11 $x = 3t, \; y = -7t, \; z = 2t, \; t$ is real

REVIEW SET 14B

1 $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **2** **a** $[10]$ **b** $\begin{bmatrix}4 & 3 & 2\\8 & 6 & 4\\0 & 0 & 0\end{bmatrix}$ **c** $[15 \;\; 18 \;\; 21]$

d CA does not exist **e** $\begin{bmatrix}5\\7\\5\end{bmatrix}$

3 **a** $\begin{bmatrix}\frac{7}{2} & -4\\\frac{-5}{2} & 3\end{bmatrix}$ **b** does not exist **c** $\begin{bmatrix}1 & \frac{5}{3}\\-2 & -\frac{11}{3}\end{bmatrix}$

4 **b** $2A - I$ **5** $56.30 **6** $AB = I$, $BA = I$, $A^{-1} = B$

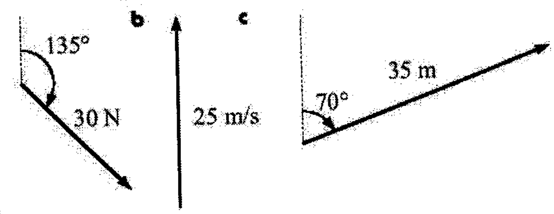
7 $x = 2, y = 1, z = 3$

8 $x = \dfrac{-13t - 1}{9}, \; y = \dfrac{20t + 14}{9}, \; z = t, \; t \in \mathcal{R}$

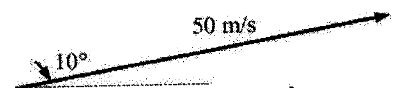
9 **b** when $m \neq \frac{14}{3}$

EXERCISE 15A.1

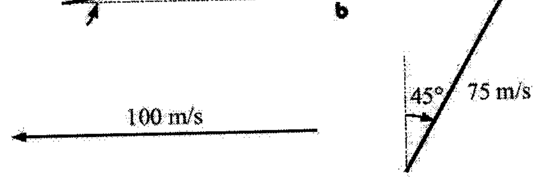
1 **a** **b** **c**



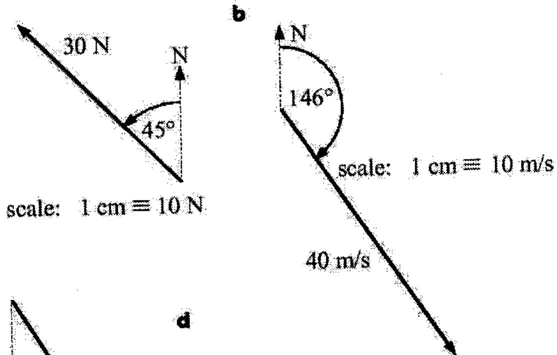
d



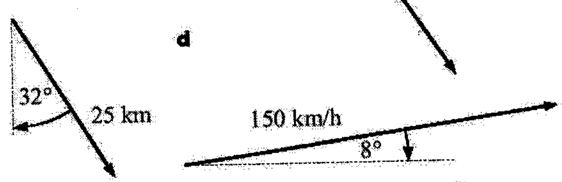
2 **a** **b**



3 **a** **b**



c **d**



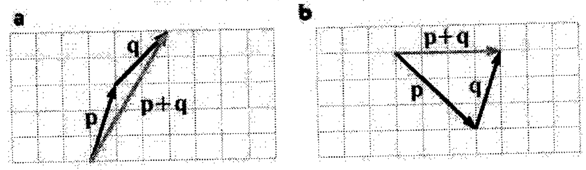
EXERCISE 15A.2

1 **a** p, q, s, t **b** p, q, r, t **c** p and r, q and t **d** q, t
e p and q, p and t

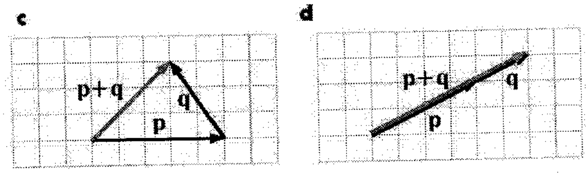
2 **a** true **b** true **c** false **d** false **e** true **f** false

EXERCISE 15B.1

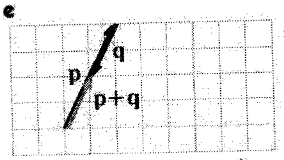
1 **a** **b**



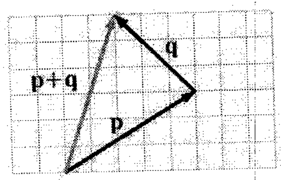
c **d**



e

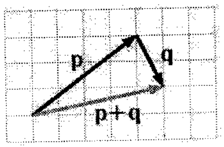


f

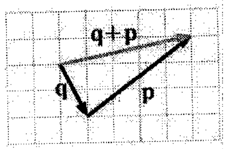


2 **a** $\overrightarrow{AC}$ **b** $\overrightarrow{BD}$ **c** $\overrightarrow{AD}$ **d** $\overrightarrow{AD}$

3 **a** **i**



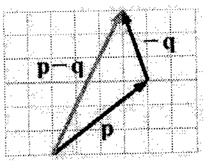
ii



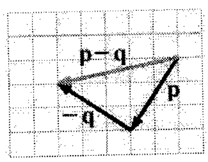
b yes

EXERCISE 15B.2

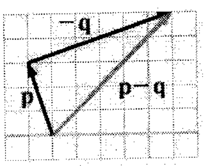
1 **a**



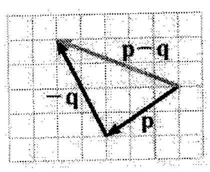
b



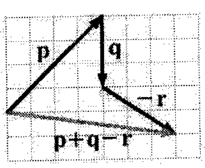
c



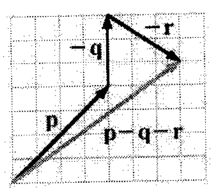
d



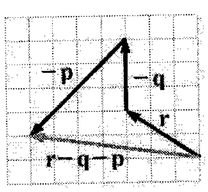
2 **a**



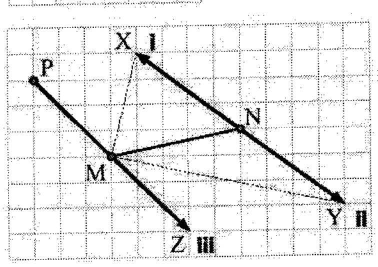
b



c



3 **a**



b a parallelogram

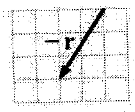
4 **a** $\overrightarrow{AB}$ **b** $\overrightarrow{AB}$ **c** 0 **d** $\overrightarrow{AD}$ **e** 0 **f** $\overrightarrow{AD}$

5 **a** t = r + s **b** r = −s − t **c** r = −p − q − s
d r = q − p + s **e** p = t + s + r − q
f p = −u + t + s − r − q

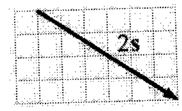
6 **a** **i** r + s **ii** −t − s **iii** r + s + t
b **i** p + q **ii** q + r **iii** p + q + r

EXERCISE 15B.3

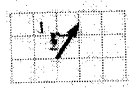
1 **a**



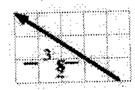
b



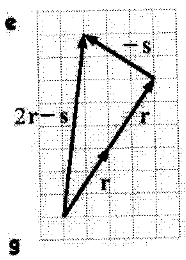
c



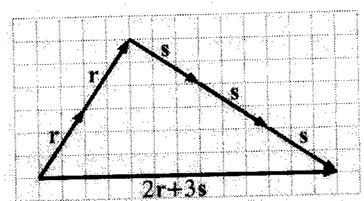
d



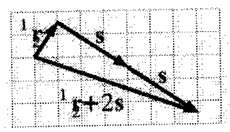
e



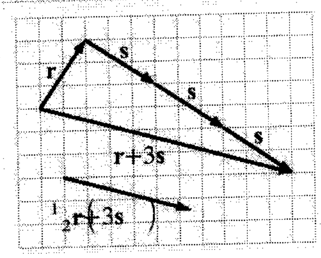
f



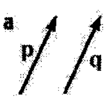
g



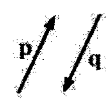
h



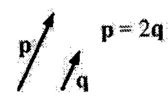
2 **a**



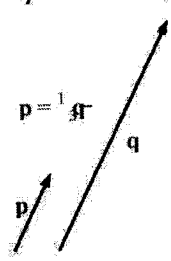
b



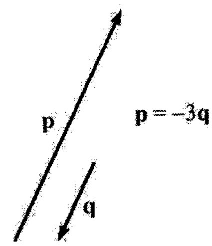
c



d

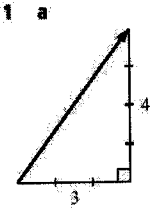


e

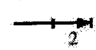


EXERCISE 15C.1

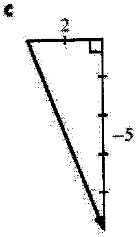
1 **a**



b



c



d

2 **a** $\begin{bmatrix}7\\3\end{bmatrix}$ **b** $\begin{bmatrix}-6\\0\end{bmatrix}$ **c** $\begin{bmatrix}2\\-5\end{bmatrix}$ **d** $\begin{bmatrix}0\\6\end{bmatrix}$ **e** $\begin{bmatrix}-6\\3\end{bmatrix}$ **f** $\begin{bmatrix}-5\\-5\end{bmatrix}$

EXERCISE 15C.2

1 **a** $\begin{bmatrix}-2\\6\end{bmatrix}$ **b** $\begin{bmatrix}-2\\6\end{bmatrix}$ **c** $\begin{bmatrix}-1\\-1\end{bmatrix}$ **d** $\begin{bmatrix}-1\\-1\end{bmatrix}$ **e** $\begin{bmatrix}-5\\-3\end{bmatrix}$
f $\begin{bmatrix}-5\\-3\end{bmatrix}$ **g** $\begin{bmatrix}-6\\4\end{bmatrix}$ **h** $\begin{bmatrix}-4\\1\end{bmatrix}$

2 **a** $\begin{bmatrix}-3\\7\end{bmatrix}$ **b** $\begin{bmatrix}-4\\-3\end{bmatrix}$ **c** $\begin{bmatrix}-8\\-1\end{bmatrix}$ **d** $\begin{bmatrix}-6\\9\end{bmatrix}$ **e** $\begin{bmatrix}0\\-5\end{bmatrix}$
f $\begin{bmatrix}6\\-9\end{bmatrix}$ **3** **a** $\begin{bmatrix}-5\\4\end{bmatrix}$ **b** $\begin{bmatrix}1\\2\end{bmatrix}$ **c** $\begin{bmatrix}6\\-5\end{bmatrix}$

EXERCISE 15C.3

1 a $\begin{bmatrix}-3\\-15\end{bmatrix}$ **b** $\begin{bmatrix}-1\\2\end{bmatrix}$ **c** $\begin{bmatrix}0\\14\end{bmatrix}$ **d** $\begin{bmatrix}5\\-3\end{bmatrix}$ **e** $\begin{bmatrix}\frac{5}{2}\\\frac{11}{2}\end{bmatrix}$ **f** $\begin{bmatrix}-7\\7\end{bmatrix}$

g $\begin{bmatrix}5\\11\end{bmatrix}$ **h** $\begin{bmatrix}3\\\frac{17}{3}\end{bmatrix}$ **2 a** $\begin{bmatrix}8\\-1\end{bmatrix}$ **b** $\begin{bmatrix}8\\-1\end{bmatrix}$ **c** $\begin{bmatrix}8\\-1\end{bmatrix}$

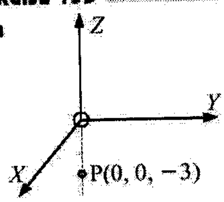
EXERCISE 15C.4

1 a $\sqrt{13}$ units **b** $\sqrt{17}$ units **c** $5\sqrt{2}$ units **d** $\sqrt{10}$ units
e $\sqrt{29}$ units

2 a $\sqrt{10}$ units **b** $2\sqrt{10}$ units **c** $2\sqrt{10}$ units **d** $3\sqrt{10}$ units
e $3\sqrt{10}$ units **f** $2\sqrt{5}$ units **g** $8\sqrt{5}$ units **h** $8\sqrt{5}$ units
i $\sqrt{5}$ units **j** $\sqrt{5}$ units

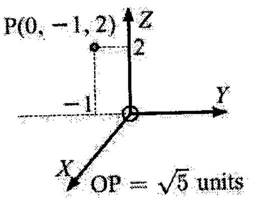
EXERCISE 15D

1 a



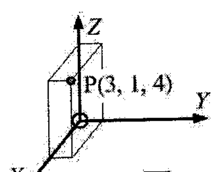
OP = 3 units

b



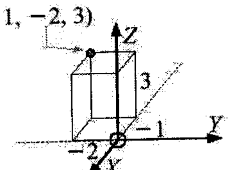
OP = $\sqrt{5}$ units

c



OP = $\sqrt{26}$ units

d P(−1, −2, 3)



OP = $\sqrt{14}$ units

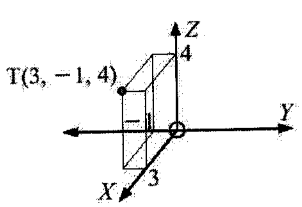
2 a i $\sqrt{14}$ units **ii** $(-\frac{1}{2}, \frac{1}{2}, 2)$ **b i** $\sqrt{14}$ units **ii** $(1, -\frac{1}{2}, \frac{3}{2})$
c i $\sqrt{21}$ units **ii** $(1, -\frac{1}{2}, 0)$ **d i** $\sqrt{14}$ units **ii** $(1, \frac{1}{2}, -\frac{3}{2})$

4 a isosceles **b** right angled **c** right angled
d straight line **5** (0, 3, 5), $r = \sqrt{3}$ units

6 a $(0, y, 0)$ **b** (0, 2, 0) and (0, −4, 0)

EXERCISE 15E.1

1 a



b $\overrightarrow{OT} = \begin{bmatrix}3\\-1\\4\end{bmatrix}$

c OT = $\sqrt{26}$ units

2 a $\overrightarrow{AB} = \begin{bmatrix}4\\-1\\-3\end{bmatrix}$, $\overrightarrow{BA} = \begin{bmatrix}-4\\1\\3\end{bmatrix}$ **b** AB = $\sqrt{26}$ units
BA = $\sqrt{26}$ units

3 $\overrightarrow{OA} = \begin{bmatrix}3\\1\\0\end{bmatrix}$, $\overrightarrow{OB} = \begin{bmatrix}-1\\1\\2\end{bmatrix}$, $\overrightarrow{AB} = \begin{bmatrix}-4\\0\\2\end{bmatrix}$

4 a $\overrightarrow{NM} = \begin{bmatrix}5\\-4\\-1\end{bmatrix}$ **b** $\overrightarrow{MN} = \begin{bmatrix}-5\\4\\1\end{bmatrix}$ **c** MN = $\sqrt{42}$ units

5 a $\overrightarrow{OA} = \begin{bmatrix}-1\\2\\5\end{bmatrix}$, OA = $\sqrt{30}$ units

b $\overrightarrow{AC} = \begin{bmatrix}-2\\-1\\-5\end{bmatrix}$, AC = $\sqrt{30}$ units

c $\overrightarrow{CB} = \begin{bmatrix}5\\-1\\3\end{bmatrix}$, CB = $\sqrt{35}$ units

6 a $\sqrt{13}$ units **b** $\sqrt{14}$ units **c** 3 units

EXERCISE 15E.2

1 a $a = 5$, $b = 6$, $c = -6$ **b** $a = 4$, $b = 2$, $c = 1$

2 a $a = \frac{1}{3}$, $b = 2$, $c = 1$ **b** $a = 1$, $b = 2$
c $a = 1$, $b = -1$, $c = 2$

3 a $r = 2$, $s = 4$, $t = -7$ **b** $r = -4$, $s = 0$, $t = 3$

4 a $\overrightarrow{AB} = \begin{bmatrix}2\\-5\\-1\end{bmatrix}$, $\overrightarrow{DC} = \begin{bmatrix}2\\-5\\-1\end{bmatrix}$
b ABCD is a parallelogram **5 a** S = (−2, 8, −3)

EXERCISE 15F

1 a $\mathbf{x} = \frac{1}{2}\mathbf{q}$ **b** $\mathbf{x} = 2\mathbf{n}$ **c** $\mathbf{x} = -\frac{1}{3}\mathbf{p}$ **d** $\mathbf{x} = \frac{1}{2}(\mathbf{r} - \mathbf{q})$
e $\mathbf{x} = \frac{1}{5}(4\mathbf{s} - \mathbf{t})$ **f** $\mathbf{x} = 3(4\mathbf{m} - \mathbf{n})$

2 a $\mathbf{y} = \begin{bmatrix}-1\\\frac{3}{2}\end{bmatrix}$ **b** $\mathbf{y} = \begin{bmatrix}2\\4\end{bmatrix}$ **c** $\mathbf{y} = \begin{bmatrix}\frac{3}{2}\\-\frac{1}{2}\end{bmatrix}$ **d** $\mathbf{y} = \begin{bmatrix}\frac{5}{4}\\\frac{3}{4}\end{bmatrix}$

4 a $\mathbf{x} = \begin{bmatrix}4\\-6\\-5\end{bmatrix}$ **b** $\mathbf{x} = \begin{bmatrix}1\\-\frac{2}{3}\\\frac{5}{3}\end{bmatrix}$ **c** $\mathbf{x} = \begin{bmatrix}\frac{3}{2}\\-1\\\frac{5}{2}\end{bmatrix}$

5 $\overrightarrow{AB} = \begin{bmatrix}3\\4\\-2\end{bmatrix}$, AB = $\sqrt{29}$ units

7 a $\overrightarrow{BD} = \frac{1}{2}\mathbf{a}$ **b** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **c** $\overrightarrow{BA} = -\mathbf{b} + \mathbf{a}$
d $\overrightarrow{OD} = \mathbf{b} + \frac{1}{2}\mathbf{a}$ **e** $\overrightarrow{AD} = \mathbf{b} - \frac{1}{2}\mathbf{a}$ **f** $\overrightarrow{DA} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

8 a $\begin{bmatrix}-1\\5\\-1\end{bmatrix}$ **b** $\begin{bmatrix}-3\\4\\-2\end{bmatrix}$ **c** $\begin{bmatrix}-3\\6\\-5\end{bmatrix}$

9 a $\begin{bmatrix}3\\1\\-2\end{bmatrix}$ **b** $\begin{bmatrix}1\\-3\\4\end{bmatrix}$ **c** $\begin{bmatrix}1\\4\\-9\end{bmatrix}$ **d** $\begin{bmatrix}2\\-4\\10\end{bmatrix}$ **e** $\begin{bmatrix}3\\2\\-5\end{bmatrix}$

f $\begin{bmatrix}-1\\\frac{3}{2}\\-\frac{7}{2}\end{bmatrix}$ **g** $\begin{bmatrix}1\\-4\\7\end{bmatrix}$ **h** $\begin{bmatrix}4\\2\\-2\end{bmatrix}$

10 a $\sqrt{11}$ units **b** $\sqrt{14}$ units **e** $\begin{bmatrix}\sqrt{11}\\-3\sqrt{11}\\2\sqrt{11}\end{bmatrix}$ **f** $\begin{bmatrix}-\frac{1}{\sqrt{11}}\\\frac{1}{\sqrt{11}}\\\frac{3}{\sqrt{11}}\end{bmatrix}$
c $\sqrt{38}$ units **d** $\sqrt{3}$ units

EXERCISE 15G

1 a M(1, 4) **b** $\overrightarrow{CA} = \begin{bmatrix}7\\5\end{bmatrix}$, $\overrightarrow{CM} = \begin{bmatrix}5\\3\end{bmatrix}$, $\overrightarrow{CB} = \begin{bmatrix}3\\1\end{bmatrix}$

2 a B(−1, 10) **b** B(−2, −9) **c** B(7, 4)

3 C(5, 1, −8), D(8, −1, −13), E(11, −3, −18)

4 a parallelogram **b** parallelogram **c** not parallelogram

5 a D(9, −1) **b** R(3, 1, 6) **c** X(2, −1, 0)

6 a $r = 2$, $s = -5$ **b** $r = 4$, $s = -1$

7 a −7 : 2 **b** −1 : 2

8 a $a = 7$, $b = -1$ **b** $a = -\frac{7}{2}$, $b = -\frac{21}{2}$

EXERCISE 15H

1 $r = 3$, $s = -9$

2 $a = -6$, $b = -4$

3 a $\begin{bmatrix}\frac{2}{3}\\-\frac{1}{3}\\-\frac{2}{3}\end{bmatrix}$ **b** $\begin{bmatrix}-\frac{4}{3}\\-\frac{2}{3}\\\frac{4}{3}\end{bmatrix}$

4 a $\overrightarrow{AB} \parallel \overrightarrow{CD}$, AB = 3CD
b $\overrightarrow{RS} \parallel \overrightarrow{KL}$, RS = $\frac{1}{2}$KL opposite direction