[07-10-29-S-Phy

EXERCISES 11.2

1–6 ■ Find a vector **a** with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

1. $A(1,3),\ \ B(4,4)$ **2.** $A(-3,4),\ \ B(-1,0)$

3. $A(3,-1),\ \ B(3,-3)$ **4.** $A(4,-1),\ \ B(1,2)$

5. $A(0,3,1),\ \ B(2,3,-1)$ **6.** $A(1,-2,0),\ \ B(1,-2,3)$

7–10 ■ Find the sum of the given vectors and illustrate geometrically.

7. $\langle 2,3\rangle,\ \ \langle 3,-4\rangle$ **8.** $\langle -1,2\rangle,\ \ \langle 5,3\rangle$

9. $\langle 1,0,1\rangle,\ \ \langle 0,0,1\rangle$ **10.** $\langle 0,3,2\rangle,\ \ \langle 1,0,-3\rangle$

11–18 ■ Find $|\mathbf{a}|$, $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $2\mathbf{a}$, and $3\mathbf{a}+4\mathbf{b}$.

11. $\mathbf{a} = \langle 5,-12\rangle,\ \ \mathbf{b} = \langle -2,8\rangle$

12. $\mathbf{a} = \langle -1,2\rangle,\ \ \mathbf{b} = \langle 4,3\rangle$

13. $\mathbf{a} = \langle 2,-3,6\rangle,\ \ \mathbf{b} = \langle 1,1,4\rangle$

14. $\mathbf{a} = \langle 3,2,-1\rangle,\ \ \mathbf{b} = \langle 0,6,7\rangle$

15. $\mathbf{a} = \mathbf{i} - \mathbf{j},\ \ \mathbf{b} = \mathbf{i} + \mathbf{j}$

16. $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j},\ \ \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}$

17. $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k},\ \ \mathbf{b} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$

18. $\mathbf{a} = 6\mathbf{i} + \mathbf{k},\ \ \mathbf{b} = \mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$

19–24 ■ Find a unit vector that has the same direction as the given vector.

19. $\langle 1,2\rangle$ **20.** $\langle 3,-5\rangle$

21. $\langle -2,4,3\rangle$ **22.** $\langle 1,-4,8\rangle$

23. $\mathbf{i} + \mathbf{j}$ **24.** $2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$

25–26 ■ Express **i** and **j** in terms of **a** and **b**.

25. $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j},\ \ \mathbf{b} = \mathbf{i} - \mathbf{j}$

26. $\mathbf{a} = \mathbf{i} - 2\mathbf{j},\ \ \mathbf{b} = 3\mathbf{i} + \mathbf{j}$

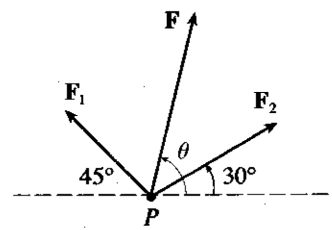
27. If A, B, and C are the vertices of a triangle, find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.

28. Let C be the point on the line segment AB that is twice as far from B as it is from A. If $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$, show that $\mathbf{c} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

29. (a) Draw the vectors $\mathbf{a} = \langle 3,2\rangle$, $\mathbf{b} = \langle 2,-1\rangle$, and $\mathbf{c} = \langle 7,1\rangle$.
(b) Show, by means of a sketch, that there are scalars s and t such that $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$.
(c) Use the sketch to estimate the values of s and t.
(d) Find the exact values of s and t.

30. Suppose that **a** and **b** are nonzero vectors that are not parallel and **c** is any vector in the plane determined by **a** and **b**. Give a geometric argument to show that **c** can be written as $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$ for suitable scalars s and t. Then give an argument using components.

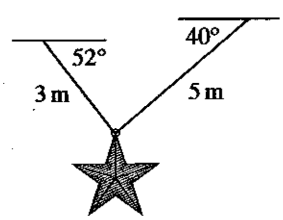
31. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ with magnitudes 10 lb and 12 lb act on an object at a point P as shown in the figure. Find the resultant force **F** acting at P as well as its magnitude and its direction. (Indicate the direction by finding the angle θ shown in the figure.)



32. Velocities have both direction and magnitude and thus are vectors. The magnitude of a velocity vector is called *speed.* Suppose that a wind is blowing from the direction N45°W at a speed of 50 km/h. (This means that the direction from which the wind blows is 45° west of the northerly direction.) A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 250 km/h. The *true course,* or *track,* of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The *ground speed* of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.

33. A woman walks due west on the deck of a ship at 3 mi/h. The ship is moving north at a speed of 22 mi/h. Find the speed and direction of the woman relative to the surface of the water.

34. Ropes 3 m and 5 m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52° and 40° with the horizontal. Find the tension in each wire and the magnitude of each tension.



35. If $\mathbf{r} = \langle x,y,z\rangle$ and $\mathbf{r}_0 = \langle x_0,y_0,z_0\rangle$, describe the set of all points (x,y,z) such that $|\mathbf{r} - \mathbf{r}_0| = 1$.

36. If $\mathbf{r} = \langle x,y\rangle$, $\mathbf{r}_1 = \langle x_1,y_1\rangle$, and $\mathbf{r}_2 = \langle x_2,y_2\rangle$, describe the set of all points (x,y) such that $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2| = k$, where $k > |\mathbf{r}_1 - \mathbf{r}_2|$.

EXERCISES 11.3

1–8 ■ Find $\mathbf{a} \cdot \mathbf{b}$.

1. $\mathbf{a} = \langle 2, 5 \rangle$, $\mathbf{b} = \langle -3, 1 \rangle$

2. $\mathbf{a} = \langle -2, -8 \rangle$, $\mathbf{b} = \langle 6, -4 \rangle$

3. $\mathbf{a} = \langle 4, 7, -1 \rangle$, $\mathbf{b} = \langle -2, 1, 4 \rangle$

4. $\mathbf{a} = \langle -1, -2, -3 \rangle$, $\mathbf{b} = \langle 2, 8, -6 \rangle$

10. (a) Show that $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$.
(b) Show that $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$.

11–16 ■ Find the angle between the vectors. (First find an exact expression and then approximate to the nearest degree.)

11. $\mathbf{a} = \langle 1, 2, 2 \rangle$, $\mathbf{b} = \langle 3, 4, 0 \rangle$

12. $\mathbf{a} = \langle 6, 0, 2 \rangle$, $\mathbf{b} = \langle 5, 3, -2 \rangle$

13. $\mathbf{a} = \langle 1, 2 \rangle$, $\mathbf{b} = \langle 12, -5 \rangle$

14. $\mathbf{a} = \langle 3, 1 \rangle$, $\mathbf{b} = \langle 2, 4 \rangle$

15. $\mathbf{a} = 6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

16. $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{j} - 3\mathbf{k}$

17–18 ■ Find, correct to the nearest degree, the three angles of the triangle with the given vertices.

17. $A(1, 2, 3)$, $B(6, 1, 5)$, $C(-1, -2, 0)$

18. $P(0, -1, 6)$, $Q(2, 1, -3)$, $R(5, 4, 2)$

19–24 ■ Determine whether the given vectors are orthogonal, parallel, or neither.

19. $\mathbf{a} = \langle 2, -4 \rangle$, $\mathbf{b} = \langle -1, 2 \rangle$

20. $\mathbf{a} = \langle 2, -4 \rangle$, $\mathbf{b} = \langle 4, 2 \rangle$

21. $\mathbf{a} = \langle 2, 8, -3 \rangle$, $\mathbf{b} = \langle -1, 2, 5 \rangle$

22. $\mathbf{a} = \langle -1, 5, 2 \rangle$, $\mathbf{b} = \langle 4, 2, -3 \rangle$

23. $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

24. $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = -3\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$

25–28 ■ Find the values of x such that the given vectors are orthogonal.

25. $x\mathbf{i} - 2\mathbf{j}$, $x\mathbf{i} + 8\mathbf{j}$

26. $x\mathbf{i} + 2x\mathbf{j}$, $x\mathbf{i} - 2\mathbf{j}$

27. $\langle x, 1, 2 \rangle$, $\langle 3, 4, x \rangle$

28. $\langle x, x, -1 \rangle$, $\langle 1, x, 6 \rangle$

29. Find a unit vector that is orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

30. For what values of c is the angle between the vectors $\langle 1, 2, 1 \rangle$ and $\langle 1, 0, c \rangle$ equal to 60°?

31–35 ■ Find the direction cosines and direction angles of the vector. (Give the direction angles correct to the nearest degree.)

31. $\langle 1, 2, 2 \rangle$

32. $\langle -4, -1, 2 \rangle$

33. $-8\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

34. $3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$

35. $\langle 2, 1.2, 0.8 \rangle$

36. If a vector has direction angles $\alpha = \pi/4$ and $\beta = \pi/3$, find the third direction angle γ.

37–42 ■ Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

37. $\mathbf{a} = \langle 2, 3 \rangle$, $\mathbf{b} = \langle 4, 1 \rangle$

38. $\mathbf{a} = \langle 3, -1 \rangle$, $\mathbf{b} = \langle 2, 3 \rangle$

39. $\mathbf{a} = \langle 4, 2, 0 \rangle$, $\mathbf{b} = \langle 1, 1, 1 \rangle$

5. $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$

6. $\mathbf{a} = \mathbf{i} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$

7. $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/3$

8. $|\mathbf{a}| = 6$, $|\mathbf{b}| = \frac{1}{3}$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/4$

9. If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, show that $\mathbf{a} \cdot \mathbf{i} = a_1$, $\mathbf{a} \cdot \mathbf{j} = a_2$, and $\mathbf{a} \cdot \mathbf{k} = a_3$.

40. $\mathbf{a} = \langle -1, -2, 2 \rangle$, $\mathbf{b} = \langle 3, 3, 4 \rangle$

41. $\mathbf{a} = \mathbf{i} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j}$

42. $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$

43. Show that the vector

$$\text{orth}_{\mathbf{a}}\,\mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}}\,\mathbf{b}$$

is orthogonal to $\mathbf{a}$. (It is called an **orthogonal projection** of $\mathbf{b}$.)

44. For the vectors in Exercise 38, find $\text{orth}_{\mathbf{a}}\,\mathbf{b}$ and illustrate by drawing the vectors $\mathbf{a}$, $\mathbf{b}$, $\text{proj}_{\mathbf{a}}\,\mathbf{b}$, and $\text{orth}_{\mathbf{a}}\,\mathbf{b}$.

45. If $\mathbf{a} = \langle 3, 0, -1 \rangle$, find a vector $\mathbf{b}$ such that $\text{comp}_{\mathbf{a}}\,\mathbf{b} = 2$.

46. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors.
(a) Under what circumstances is $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \text{comp}_{\mathbf{b}}\,\mathbf{a}$?
(b) Under what circumstances is $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \text{proj}_{\mathbf{b}}\,\mathbf{a}$?

47. A constant force with vector representation $\mathbf{F} = 10\mathbf{i} + 18\mathbf{j} - 6\mathbf{k}$ moves an object along a straight line from the point $(2, 3, 0)$ to the point $(4, 9, 15)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.

48. Find the work done by a force of 20 lb acting in the direction N50°W in moving an object 4 ft due west.

49. A woman exerts a horizontal force of 25 lb on a crate as she pushes it up a ramp that is 10 ft long and inclined at an angle of 20° above the horizontal. Find the work done on the box.

50. A wagon is pulled a distance of 100 m along a horizontal path by a constant force of 50 N. The handle of the wagon is at an angle of 30° above the horizontal. How much work is done?

51. Which of the following expressions have no meaning?
(a) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ (b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$
(c) $|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})$ (d) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$
(e) $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$ (f) $|\mathbf{a}| \cdot (\mathbf{b} + \mathbf{c})$

52. Suppose that all sides of a quadrilateral are equal in length and opposite sides are parallel. Use vector methods to show that the diagonals are perpendicular.

53. Use a scalar projection to show that the distance from a point $P_1(x_1, y_1)$ to the line with equation $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Use this formula to find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$.

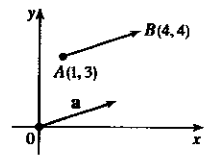
54. If $\mathbf{r} = \langle x, y, z \rangle$, $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, show that the vector equation $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$ represents a sphere, and find its center and radius.

55. Find the angle between a diagonal of a cube and one of its edges.

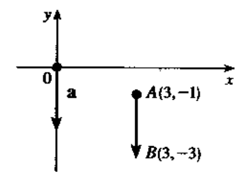
56. Find the angle between a diagonal of a cube and a diagonal of one of its faces.

Exercises 11.2 ■ page 675

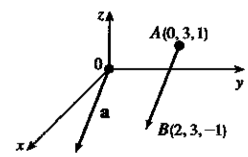
1. $\mathbf{a} = \langle 3, 1 \rangle$



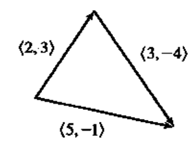
3. $\mathbf{a} = \langle 0, -2 \rangle$



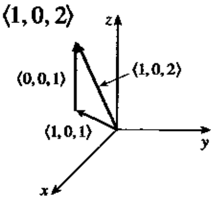
5. $\mathbf{a} = \langle 2, 0, -2 \rangle$



7. $\langle 5, -1 \rangle$

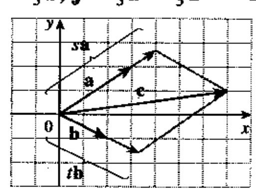


9. $\langle 1, 0, 2 \rangle$



11. $13, \langle 3, -4 \rangle, \langle 7, -20 \rangle, \langle 10, -24 \rangle, \langle 7, -4 \rangle$

13. $7, \langle 3, -2, 10 \rangle, \langle 1, -4, 2 \rangle, \langle 4, -6, 12 \rangle, \langle 10, -5, 34 \rangle$
15. $\sqrt{2}, 2\mathbf{i}, -2\mathbf{j}, 2\mathbf{i} - 2\mathbf{j}, 7\mathbf{i} + \mathbf{j}$
17. $\sqrt{3}, 3\mathbf{i} + 4\mathbf{k}, -\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}, 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, 11\mathbf{i} - \mathbf{j} + 15\mathbf{k}$ **19.** $\langle 1/\sqrt{5}, 2/\sqrt{5} \rangle$
21. $\langle -2/\sqrt{29}, 4/\sqrt{29}, 3/\sqrt{29} \rangle$ **23.** $(\mathbf{i} + \mathbf{j})/\sqrt{2}$
25. $\mathbf{i} = \frac{1}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}, \mathbf{j} = \frac{1}{5}\mathbf{a} - \frac{2}{5}\mathbf{b}$ **27.** $\mathbf{0}$
29. (a), (b)



(d) $s = \frac{9}{7}, t = \frac{11}{7}$

31. $\mathbf{F} = (6\sqrt{3} - 5\sqrt{2})\mathbf{i} + (6 + 5\sqrt{2})\mathbf{j} \approx 3.32\mathbf{i} + 13.07\mathbf{j}$, $|\mathbf{F}| \approx 13.5$ lb, $\theta \approx 76°$ **33.** $\sqrt{493} \approx 22.2$ mi/h N8°W
35. A sphere with radius 1, centered at (x_0, y_0, z_0)

Exercises 11.3 ■ page 680

1. -1 **3.** -5 **5.** -11 **7.** 3 **11.** $\cos^{-1}\frac{11}{15} \approx 43°$
13. $\cos^{-1}(2/(13\sqrt{5})) \approx 86°$ **15.** $\cos^{-1}(1/(7\sqrt{3})) \approx 85°$
17. 114°, 33°, 33° **19.** Parallel **21.** Neither
23. Orthogonal **25.** ± 4 **27.** $-\frac{4}{5}$
29. $(\mathbf{i} - \mathbf{j} - \mathbf{k})/\sqrt{3}$ [or $(-\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}$]
31. $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$; 71°, 48°, 48°
33. $-8/\sqrt{77}, 3/\sqrt{77}, 2/\sqrt{77}$; 156°, 70°, 77°
35. $5/\sqrt{38}, 3/\sqrt{38}, 2/\sqrt{38}$; 36°, 61°, 71°
37. $11/\sqrt{13}, \langle 22/13, 33/13 \rangle$ **39.** $3/\sqrt{5}, \langle 6/5, 3/5, 0 \rangle$
41. $1/\sqrt{2}, (\mathbf{i} + \mathbf{k})/2$
45. $\langle 0, 0, -2\sqrt{10} \rangle$ or any vector of the form $\langle s, t, 3s - 2\sqrt{10} \rangle$, $s, t \in \mathbb{R}$ **47.** 38 J **49.** $250 \cos 20° \approx 235$ ft-lb
51. (a), (e), (f) **53.** $\frac{13}{5}$ **55.** $\cos^{-1}(1/\sqrt{3}) \approx 55°$