

3.5 Exercises

Factor out the greatest common factor. See Examples 1–6.

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| –1. $15k + 30$ | 2. $7m - 21$ |
| 3. $6p^2 + 4p$ | 4. $12a^3 + 9a^2$ |
| –5. $8m^2p - 4mp^2$ | 6. $12z^2 - 6zw^3$ |
| 7. $5xy - 8xy^2$ | 8. $4ab + 2a^2b$ |
| –9. $8m^4 + 6m^3 - 4m^2$ | 10. $2p^5 + 4p^6 - 8p^3$ |
| 11. $10t^5 - 8t^4 - 16t^3$ | 12. $6p^3 - 18p^2 + 9p^4$ |
| –13. $6r^2s - 3rs^2 - 9r^3s$ | 14. $5p^3t^2 + 25p^2t^3 + 10p^3t^4$ |
| 15. $2x^2y - 3xy^2 + 4x^2y^2$ | 16. $14a^3b^2 + 7a^2b - 21a^5b^3$ |
| –17. $12km^3 - 24k^3m^2 + 36k^2m^4$ | 18. $15m^3p^3 - 6m^3p^4 + 9mp^3$ |
| 19. $144z^{11}m^4 + 16z^3m^5 - 32z^4m^5$ | 20. $39a^5b^3 - 26a^7b^2 + 52a^8b^5$ |
| –21. $16z^2n^6 + 64zn^7 - 32z^3n^3$ | 22. $5r^3s^5 + 10r^2s^2 - 15r^4s^2$ |
| 23. $15a^3b + 12a^2c - 3ad^4$ | 24. $20m^4y + 30m^2y^2 + 50m^3z^2$ |
| –25. $9x^2y^2z^4 - 18x^3y^2z$ | 26. $30m^5p^5q^3 + 20m^6p^4q^2$ |
| 27. $12p^5q^6 + 5r^2s^2 + 24p^3r^2$ | 28. $2m^8k^8 + 7q^4p^5 + 14m^6p^3$ |
| –29. $14a^3b^2 + 7a^2b - 21a^5b^3 + 42ab^4$ | 30. $12km^3 - 24k^3m^2 + 36k^2m^4 - 60k^4m^3$ |
| 31. $-15m^3p^3 - 6m^3p^4 - 9mp^3 + 30m^2p^3$ | 32. $-25x^3y^2 - 20x^4y^3 + 15x^5y^4 - 50x^6y^2$ |
| –33. $(m - 9)(m + 1) + (m - 9)(m + 2)$ | 34. $(a + 5)(a - 6) + (a + 5)(a - 1)$ |
| 35. $(3k - 7)(k + 2) + (3k - 7)(k + 5)$ | 36. $(5m - 11)(2m + 5) + (5m - 11)(m + 5)$ |
| –37. $(r - 6)(2r + 1) - (r - 6)(r + 3)$ | 38. $(y + 2)(2y + 3) - (y + 2)(y + 1)$ |
| 39. $m^5(r + s) + m^5(t + u)$ | $\left\{ \begin{array}{l} 40. z^3(k + m) + z^3(p + q) \\ 42. 2(t - s) + 4(t - s)^2 - (t - s)^3 \\ 44. 6(a + 2b)^2 - 4(a + 2b)^3 + 12(a + 2b)^4 \\ 46. -9a^2(p + q) - 3a^3(p + q)^2 + 6a(p + q)^3 \end{array} \right.$ |
| 41. $4(3 - x)^2 - (3 - x)^3 + 3(3 - x)$ | |
| 43. $15(2z + 1)^3 + 10(2z + 1)^2 - 25(2z + 1)$ | |
| 45. $5(m + p)^3 - 10(m + p)^2 - 15(m + p)^4$ | |

Factor each of the following polynomials twice. First, use a common factor with a positive coefficient, and then use a common factor with a negative coefficient. See Example 6.

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| 47. $24z^2 - 48z$ | 48. $39p^3 - 65p^4$ |
| 49. $-2x^5 + 6x^3 + 4x^2$ | 50. $-5a^3 + 10a^4 - 15a^5$ |
| 51. $-32a^4m^5 - 16a^2m^3 - 64a^5m^6$ | 52. $-144z^{11}n^5 + 16z^3n^{11} - 32z^4n^7$ |

Factor by grouping. See Examples 7–9.

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|-----------------------------------|-----------------------------------|
| 53. $pq + 3rq + pm + 3rm$ | 54. $5x + 5a + 9bx + 9ab$ |
| 55. $2b + 2c + ab + ac$ | 56. $3am + 3ap + 2bm + 2bp$ |
| 57. $p^2 + pq - 3py - 3yq$ | 58. $r^2 + 6rs - 3rt - 18st$ |
| 59. $a^2b^2 + 2b^2 - 5a^2 - 10$ | 60. $m^2r^2 + 8r^2 - 3m^2 - 24$ |
| 61. $x^2 - 3x + 2x - 6$ | 62. $y^2 - 8y + 4y - 32$ |
| 63. $3r^2 - 2r + 15r - 10$ | 64. $2a^2 - 6a + 7a - 21$ |
| 65. $21y^2 + 14y - 15y - 10$ | 66. $18q^2 + 9q - 4q - 2$ |
| 67. $16p^2 + 6pq - 8pq - 3q^2$ | 68. $8r^2 + 6rs - 12rs - 9s^2$ |
| 69. $14m^2 + 21mq - 2mq - 3q^2$ | 70. $10y^2 + 4yz - 5y - 2z$ |
| 71. $3a^3 + 3ab^2 + 2a^2b + 2b^3$ | 72. $16m^3 - 4m^2p^2 - 4mp + p^3$ |
| 73. $1 - a + ab - b$ | 74. $2ab^2 - 8b^2 + a - 4$ |
| 75. $8 - 6y^3 - 12y + 9y^4$ | 76. $x^3y^2 + x^3 - 3y^2 - 3$ |

Factor each of the following polynomials. Assume that all variables used as exponents represent positive integers.

77. $p^{6m} - 2p^{4m}$

79. $q^{3k} + 2q^{2k} + 3q^k$

81. $y^{r+5} + y^{r+4} + y^{r+2}$

83. $(3k - 7)(k + 2) - (7 - 3k)(2k + 1)$

85. $r^p m^p + q^p m^p - r^p z^p - q^p z^p$

78. $4r^{3z} + 8r^{5z}$

80. $z^{2x} - z^x + z^{3x}$

82. $8k^{2z+3} + 2k^{2z+1} + 12k^{2z}$

84. $(5z + 1)(3z - 5) + (2z - 7)(5 - 3z)$

86. $6a^2 b^z - 10b^z - 3a^2 c^z + 5c^z$

Factor out the greatest common factor.

87. $3m^{-5} + m^{-3}$

89. $3p^{-3} + 2p^{-2} - 4p^{-1}$

88. $k^{-2} + 2k^{-4}$

90. $-5y^{-3} + 8y^{-2} + y^{-1}$

Review Exercises Find each product by the FOIL method. See Section 3.4.

91. $(2m - 1)(3m + 2)$

92. $(4p + 5)(3p - 7)$

93. $(8r + 5s)(2r - 3s)$

94. $(7y - 2x)(3y + 5x)$

95. $(9z - 7a)(2z + 5a)$

96. $(8p + 3q)(4p - q)$

97. $(2r - 5s)(7r + 3s)$

98. $(2a + 7b)(3a + 5b)$

3.6 Exercises

Factor each of the following trinomials. See Examples 1–9.

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| 1. $c^2 + 4c - 5$ | 2. $d^2 + 9d + 8$ | 3. $p^2 + 6p + 8$ | 4. $m^2 + 15m + 56$ |
| 5. $a^2 - a - 12$ | 6. $z^2 + 2z - 35$ | 7. $r^2 - r - 20$ | 8. $y^2 - 2y - 35$ |
| 9. $x^2 - 3x - 40$ | 10. $a^2 - 6a - 16$ | 11. $k^2 - kn - 6n^2$ | 12. $a^2 + 3ab - 18b^2$ |
| 13. $y^2 - 3yx - 10x^2$ | 14. $p^2 - 2pq - 15q^2$ | 15. $a^2b^2 - 7ab + 12$ | 16. $y^2w^2 + 4yw - 21$ |
| 17. $5y^2 + y - 6$ | 18. $2r^2 - r - 3$ | 19. $3m^2 + 7m + 2$ | 20. $3y^2 + 14y + 8$ |
| 21. $8y^2 + 13y - 6$ | 22. $6x^2 + 13x + 6$ | 23. $18x^2 - 3x - 10$ | 24. $12m^2 - 8m - 15$ |
| 25. $35p^2 - 4p - 15$ | 26. $6m^2 - 17m - 14$ | 27. $12a^2 + 8ab - 15b^2$ | 28. $3m^2 + 7mk + 2k^2$ |
| 29. $4k^2 - 12ka + 9a^2$ | 30. $18a^2 - 3ab - 28b^2$ | 31. $35x^2 - 41xy - 24y^2$ | 32. $10a^2 + ab - 3b^2$ |
| 33. $8m^2 - 14mp - 39p^2$ | 34. $6x^2 - 5xy - 39y^2$ | 35. $6k^2p^2 + 13kp + 6$ | 36. $15z^2x^2 - 22zx - 5$ |
| 37. $12m^2 + 14m - 40$ | 38. $36t^2 + 30t - 50$ | 39. $18a^2 - 15a - 18$ | 40. $100r^2 - 90r + 20$ |
| 41. $6a^3 + 12a^2 - 90a$ | 42. $3m^4 + 6m^3 - 72m^2$ | 43. $13y^3 + 39y^2 - 52y$ | 44. $4p^3 + 24p^2 - 64p$ |
| 45. $2x^3y^3 - 48x^2y^4 + 288xy^5$ | | 46. $6m^3n^2 - 24m^2n^3 - 30mn^4$ | |

Factor each of the following. See Examples 10 and 11.

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| 47. $3x^4 - 14x^2 - 5$ | 48. $3p^4 - 8p^2 - 3$ | 49. $z^4 - 7z^2 - 30$ |
| 50. $k^4 + k^2 - 12$ | 51. $6x^4 + 5x^2 - 25$ | 52. $6a^4 - 11a^2 - 10$ |

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53. $12p^4 + 28p^2r - 5r^2$
 55. $4x^4 + 33x^2a^2 - 27a^4$

Find a polynomial that can be factored as follows.

57. $(3q + 7h)(q - 2h)$
 59. $-9a(a - 5b)(2a + 7b)$

Factor each of the following. Assume that all variables used as exponents represent positive integers. See Examples 10 and 11.

61. $6(p + 3)^2 + 13(p + 3) + 5$
 63. $6(z + k)^2 - 7(z + k) - 5$
 65. $a^2(a + b)^2 - ab(a + b)^2 - 6b^2(a + b)^2$
 67. $p^2(p + q) + 4pq(p + q) + 3q^2(p + q)$
 69. $z^2(z - x) - zx(x - z) - 2x^2(z - x)$
 71. $p^{2n} - p^n - 6$
 73. $6z^{4r} - 5z^{2r} - 4$
 75. $36k^{3r} + 30k^{2r} + 4k^r$

Review Exercises Simplify. See Section 3.2.

77. -5^0 78. $\left(\frac{5}{4}\right)^{-2}$

Find each product. See Section 3.4.

81. $(2m - 5)(2m + 5)$ 82. $(3p + 2q)(3p - 2q)$ 83. $(5a - 3b)^2$
 84. $(2z + 5x)^2$ 85. $(y - 2)(y^2 + 2y + 4)$ 86. $(5a + 3)(25a^2 - 15a + 9)$

54. $2y^4 + xy^2 - 6x^2$
 56. $2p^4 + 31p^2q^2 - 16q^4$

58. $(5p - 2q)(3p + 4q)$
 60. $12z^2(5z + x)(2z - x)$

62. $10(m - 5)^2 - 9(m - 5) - 9$
 64. $3(r + m)^2 - 10(r + m) - 25$
 66. $m^2(m - p) + mp(m - p) - 2p^2(m - p)$
 68. $2k^2(5 - y) - 7k(5 - y) + 5(5 - y)$
 70. $r^2(r - s) - 5rs(s - r) - 6s^2(r - s)$
 72. $k^{2y} + 4k^y - 5$
 74. $12a^{4p} + 11a^{2p} + 2$
 76. $30y^{7a} - 26y^{6a} - 40y^{5a}$

79. $\frac{(m^2p)^{-1}}{mp^2}$ 80. $\frac{(2rs)^3}{(r^{-1}s)^2}$

3.7 Exercises

Factor each of the following. See Examples 1–4.

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| 1. $x^2 - 25$ | 2. $9 - p^2$ | 3. $36m^2 - 25$ |
| 4. $4x^2 - 49$ | 5. $16y^2 - 81q^2$ | 6. $9m^2 - 100r^2$ |
| 7. $16 - 25a^2b^2$ | 8. $49 - 64x^2z^2$ | 9. $a^4 - 4b^4$ |
| 10. $m^2p^2 - 49r^2s^2$ | 11. $x^2 + 4x + 4$ | 12. $y^2 + 6y + 9$ |
| 13. $a^2 - 10a + 25$ | 14. $b^2 - 8b + 16$ | 15. $9r^2 - 6rs + s^2$ |
| 16. $4a^2 - 20ab + 25b^2$ | 17. $25x^2y^2 - 20xy + 4$ | 18. $9k^2q^2 + 24kq + 16$ |
| 19. $72m^2 - 120mp + 50p^2$ | 20. $100y^2 - 100yz + 25z^2$ | 21. $8a^3 + 1$ |
| 22. $125a^3 - 1$ | 23. $27x^3 - 64y^3$ | 24. $8a^3 + 125m^3$ |
| 25. $64x^3 + 125y^3$ | 26. $216z^3 - x^3$ | 27. $125m^3 - 8p^3$ |
| 28. $64y^3 - 1331x^3$ | 29. $1000 + 27r^3s^3$ | 30. $343 + 1000a^3b^3$ |
| 31. $64v^6 + 1$ | 32. $m^6 - 8$ | |

Factor each of the following.

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| 33. $(x + y)^2 - 16$ | 34. $(a + b)^2 - 100$ | 35. $25 - (r + 3s)^2$ |
| 36. $81 - (2k + z)^2$ | 37. $m^2 - (3p - 5)^2$ | 38. $w^2 - (2z - 3)^2$ |
| 39. $(a + b)^2 - (a - b)^2$ | 40. $(c - d)^2 - (c + d)^2$ | 41. $(a + b)^2 + 2(a + b) + 1$ |
| 42. $(x + y)^2 + 6(x + y) + 9$ | 43. $(m - p)^2 + 4(m - p) + 4$ | 44. $(w - r)^2 + 8(w - r) + 16$ |
| 45. $p^2 - 6p + 9 - r^2$ | 46. $k^2 - 10k + 25 - z^2$ | 47. $9y^2 - 30y + 25 - 16x^2$ |
| 48. $25a^2 - 20a + 4 - 9b^2$ | 49. $r^2 - 16s^2 + 24s - 9$ | 50. $t^2 - 16u^2 + 8u - 1$ |
| 51. $64 - (a - b)^3$ | 52. $(r + 1)^3 - 1$ | 53. $(p - 5)^3 + 125$ |
| 54. $m^3 + (m + 3)^3$ | 55. $a^3 - (a - 4)^3$ | 56. $(p + q)^3 - (p - q)^3$ |

Find a value of b or c so that the following will be perfect squares.

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| 57. $p^2 + 6p + c$ | 58. $y^2 - 14y + c$ | 59. $9z^2 - 30z + c$ |
| 60. $16r^2 + 24r + c$ | 61. $16q^2 + bq + 25$ | 62. $36x^2 + bx + 25$ |

Factor each of the following. Assume that all variables used as exponents represent positive integers.

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| 63. $16m^{4x} - 9$ | 64. $100m^{2q} - 81$ | 65. $64r^{8z} - 1$ |
| 66. $4 - 49x^{4y}$ | 67. $100m^{2z} - 9p^{8z}$ | 68. $16k^{8b} - 25m^{4b}$ |
| 69. $9a^{4z} - 30a^{2z} + 25$ | 70. $121p^{8k} + 44p^{4k} + 4$ | 71. $x^{3n} - 8$ |
| 72. $216 + b^{3k}$ | 73. $27z^{12y} + 125x^{6y}$ | 74. $1000k^{15r} - m^{21r}$ |

Review Exercises Factor completely. See Sections 3.5 and 3.6.

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|--------------------------------|---------------------------|------------------------|
| 75. $16y^2 - 24y + 32y^3$ | 76. $9z^2 - 10z^5 + z^7$ | 77. $xy + 2y + 4x + 8$ |
| 78. $a^2b^2 + 3b^2 + 2a^2 + 6$ | 79. $y^2 + y - 2$ | 80. $m^2 - 4m - 21$ |
| 81. $6r^2 + 19rz - 7z^2$ | 82. $10w^2 + 19wx + 6x^2$ | |