Exercises 2.5

[05-12-16-MT-11-C]

Use the identities developed in this section to evaluate the following expressions.

1. $\sin \frac{\pi}{12}$
2. $\sin \frac{\pi}{8}$
3. $\cos \frac{\pi}{8}$
4. $\sin \frac{\pi}{24}$
5. $\cos \frac{\pi}{24}$
6. $\cos \frac{\pi}{16}$
7. $\sin \frac{\pi}{16}$
8. $\tan \frac{\pi}{8}$
9. $\tan \frac{\pi}{12}$
10. $\tan \frac{\pi}{16}$
11. $\tan \frac{5\pi}{12}$
12. $\sin \frac{3\pi}{8}$
13. $\cos \frac{3\pi}{8}$
14. $\cos \frac{7\pi}{8}$
15. $\sin \frac{7\pi}{8}$
16. $\sin \frac{11\pi}{12}$
17. $\cos \frac{11\pi}{12}$
18. $\tan \frac{3\pi}{8}$
19. $\tan \frac{7\pi}{8}$
20. $\tan \frac{11\pi}{12}$

Verify that each of the following equations is an identity.

21. $\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \equiv \frac{1}{2} \sin \alpha.$
22. $\cos^2 \frac{\theta}{4} - \cos \frac{\theta}{2} \equiv \sin^2 \frac{\theta}{4}.$
23. $\tan \frac{\beta}{2} \equiv \frac{1 - \cos \beta}{\sin \beta}.$
24. $\tan \frac{\gamma}{2} \equiv \frac{\sin \gamma}{1 + \cos \gamma}.$
25. $\cot \frac{x}{2} \equiv \frac{1 + \cos x}{\sin x}.$
26. $\cot \frac{z}{2} \equiv \frac{\sin z}{1 - \cos z}.$
27. $\tan \frac{y}{2} \equiv \csc y - \cot y.$
28. $\sec^2 \frac{\phi}{2} \equiv \frac{2}{1 + \cos \phi}.$
29. $\sin \alpha \cot \frac{\alpha}{2} \equiv 2 \cos^2 \frac{\alpha}{2}.$
30. $\left(\sin \frac{y}{2} + \cos \frac{y}{2}\right)^2 \equiv 1 + \sin y.$
31. $\tan^2 \frac{\beta}{2} + 1 \equiv 2 \csc \beta \tan \frac{\beta}{2}.$
32. $\csc^2 \frac{x}{2} \equiv \frac{2}{1 - \cos x}.$
33. $\tan \frac{\phi}{2} \sin \phi \equiv 2 \sin^2 \frac{\phi}{2}.$
34. $\frac{1 + \tan \frac{z}{2}}{1 - \tan \frac{z}{2}} \equiv \sec z + \tan z.$
35. $2 \tan \frac{\alpha}{2} \csc \alpha \equiv \sec^2 \frac{\alpha}{2}.$
36. $\cot y + \csc y \equiv \cot \frac{y}{2}.$
37. $\csc (\alpha + \beta) - \cot (\alpha + \beta) \equiv \tan \left(\frac{\alpha + \beta}{2}\right).$
38. $2 \sin^2 \frac{y}{6} - \sin^2 \frac{y}{7} \equiv \cos^2 \frac{y}{7} - \cos \frac{y}{3}.$
39. $\cos^2 \frac{x}{18} \equiv \sin^2 \frac{x}{18} + \cos \frac{x}{9}.$
40. $\frac{1 - \tan \frac{\alpha}{2}}{1 + \tan \frac{\alpha}{2}} \equiv \frac{\cos \alpha}{1 + \sin \alpha}.$