Exercises 2.4

[05-12-16-MT-11-B]

Using $\cos \pi/6 = \sqrt{3}/2$, $\sin \pi/6 = 1/2$, and $\tan \pi/6 = 1/\sqrt{3}$ and the double-angle formulas, evaluate the following.

1. $\sin \frac{\pi}{3}$ 2. $\cos \frac{\pi}{3}$ 3. $\tan \frac{\pi}{3}$

Using $\cos \pi/4 = \sqrt{2}/2 = \sin \pi/4$ and the double-angle formulas, evaluate the following.

4. $\sin \frac{\pi}{2}$ 5. $\cos \frac{\pi}{2}$

Using $\sin \pi/2 = 1$ and $\cos \pi/2 = 0$, evaluate the following.

6. $\sin \pi$ 7. $\cos \pi$

Verify that each of the following equations is an identity.

8. $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.
9. $\sin 4\phi \equiv 4 \cos \phi (\sin \phi - 2 \sin^3 \phi)$.
10. $\cos 4\alpha \equiv 8 \cos^4 \alpha - 8 \cos^2 \alpha + 1$.
11. $\sin 5x \equiv 16 \sin^5 x - 20 \sin^3 x + 5 \sin x$.
12. $\cos 5y \equiv 16 \cos^5 y - 20 \cos^3 y + 5 \cos y$.
13. $\sin \frac{x}{2} \cos \frac{x}{2} \equiv \frac{1}{2} \sin x$. $\left[\textit{Hint:}\quad x = 2\left(\frac{x}{2}\right)\right]$
14. $\cos^2 \frac{z}{2} - \sin^2 \frac{z}{2} \equiv \cos z$.
15. $\left(\cos \frac{\beta}{2} - \sin \frac{\beta}{2}\right)^2 \equiv 1 - \sin \beta$.
16. $\sin 2\alpha \equiv \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$.
17. $\tan 3\phi \equiv \frac{3 \tan \phi - \tan^3 \phi}{1 - 3 \tan^2 \phi}$.
18. $\tan 2y \equiv \frac{2}{\cot y - \tan y}$.
19. $\cos 2x \equiv \cos^4 x - \sin^4 x$.
20. $1 - 2 \sin^2 \left(\frac{\pi}{4} - \theta\right) \equiv \sin 2\theta$.
21. $\cos 2z + 2 \sin^2 z \equiv 1$.
22. $\sin^2 \frac{\beta}{2} \equiv \frac{1 - \cos \beta}{2}$.
23. $\cos^2 2\gamma - 2 \cos^2 \gamma \equiv -1$.
24. $\sec 2\phi \equiv \frac{\sec^2 \phi}{1 - \tan^2 \phi}$.
25. $\sin 6 \sin 3\theta \equiv \sin^2 2\theta - \sin^2 \theta$.
26. $\cot 2y \equiv \frac{\cot^2 y - 1}{2 \cot y}$.
27. $\sec y \csc y \equiv 2 \csc 2y$.
28. $\cos 4\alpha \equiv 1 - 8 \sin^2 \alpha \cos^2 \alpha$.
29. $\cos 4z \equiv 8 \sin^4 z - 8 \sin^2 z + 1$.
30. $(\cos y - \sin y)^2 \equiv 1 - \sin 2y$.
31. $2 \sin (\alpha + \beta) \cdot \cos (\alpha - \beta) \equiv \sin 2\alpha + \sin 2\beta$.
32. $\frac{\sin 3\phi}{\sin \phi} - \frac{\cos 3\phi}{\cos \phi} \equiv 2$.
33. $2 \csc 2x \equiv \cot x + \tan x$.
34. $\cot \alpha - \cot 2\alpha \equiv \csc 2\alpha$.
35. $\csc^2 2y - \sec^2 2y \equiv 4 \cot 4y \csc 4y$.
36. $\cos \theta \cos 3\theta \equiv \cos^2 \theta - \sin^2 2\theta$.
37. $\tan \beta \equiv \frac{\sin 2\beta}{1 + \cos 2\beta}$.
38. $\cot^2 \phi - \tan^2 \phi \equiv \frac{4 \cos 2\phi}{\sin^2 2\phi}$.
39. $4 \sin^4 \gamma \equiv 3 - 4 \cos 2\gamma + \cos 4\gamma$.
40. $\frac{1 + \sin 2x + \cos 2x}{1 + \sin 2x - \cos 2x} \equiv \cot x$.