

## Exercises 2.3

[05-12-16-MT-11-A]

~~1.~~  $\sin \frac{\pi}{12}$      ~~2.~~  $\sin \frac{5\pi}{12}$      ~~3.~~  $\cos \frac{5\pi}{12}$      4.  $\sin \frac{11\pi}{12}$   
~~5.~~  $\cos \frac{11\pi}{12}$      ~~6.~~  $\cos \frac{7\pi}{12}$      ~~7.~~  $\sin \frac{7\pi}{12}$

Verify that each of the following equations is an identity.

8.  $\cos\left(\frac{\pi}{6} + x\right) \equiv \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x.$

9.  $\sin\left(x + \frac{\pi}{4}\right) \equiv \frac{\sqrt{2}}{2}(\sin x + \cos x).$

10.  $\sin\left(\theta + \frac{\pi}{2}\right) \equiv \cos \theta.$

11.  $\cos\left(\theta + \frac{\pi}{2}\right) \equiv -\sin \theta.$

12.  $\sin(\theta + \pi) \equiv -\sin \theta.$

13.  $\cos(\theta + \pi) \equiv -\cos \theta.$

14.  $\cos 4\theta \equiv \cos 3\theta \cos \theta - \sin 3\theta \sin \theta.$

15.  $\sin 7x \equiv \sin 4x \cos 3x + \cos 4x \sin 3x.$

16.  $\sin 3\phi \equiv \sin 5\phi \cos 2\phi - \cos 5\phi \sin 2\phi.$

17.  $\cos 5y \equiv \cos 9y \cos 4y + \sin 9y \sin 4y.$

18.  $\sin(x + \pi) - \sin(x - \pi) \equiv 0.$

19.  $(\sin x + \cos \beta)^2 + (\cos x + \sin \beta)^2 \equiv 2[\sin(x + \beta) + 1].$

20.  $1 - \tan \theta \tan \phi \equiv \frac{\cos(\theta + \phi)}{\cos \theta \cos \phi}.$

21.  $\cos(\alpha + \beta) \cos(\alpha - \beta) \equiv (\cos \alpha \cos \beta)^2 - (\sin \alpha \sin \beta)^2.$

22.  $(\cos \alpha \cos \beta)^2 - (\sin \alpha \sin \beta)^2 \equiv \cos^2 \alpha - \sin^2 \beta.$

23.  $\tan(\alpha + \beta) \equiv \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$  [Use  $\sin(\alpha + \beta)$  and  $\cos(\alpha + \beta)$ .]

24.  $\cot(\theta + \phi) \equiv \frac{\cot \theta \cot \phi - 1}{\cot \phi + \cot \theta}.$

25.  $\sec(x + y) \equiv \frac{\sec x \sec y}{1 - \tan x \tan y}.$

26.  $\csc(\alpha + \beta) \equiv \frac{\csc \alpha \csc \beta}{\cot \beta + \cot \alpha}.$

27.  $\tan(\theta - \phi) \equiv \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}.$

28.  $\cot(x - y) \equiv \frac{\cot x \cot y + 1}{\cot y - \cot x}.$

29.  $\sec(\alpha - \beta) \equiv \frac{\sec \alpha \sec \beta}{1 + \tan \alpha \tan \beta}.$

30.  $\csc(\theta - \phi) \equiv \frac{\csc \theta \csc \phi}{\cot \phi - \cot \theta}.$

31.  $\tan\left(\theta + \frac{\pi}{2}\right) \equiv -\cot \theta.$

32.  $\cot\left(\alpha + \frac{\pi}{2}\right) \equiv -\tan \alpha.$

33.  $\sin 2\phi \equiv 2 \sin \phi \cos \phi.$

34.  $\cos 2x \equiv \cos^2 x - \sin^2 x.$

35.  $\cos 2y \equiv 2 \cos^2 y - 1.$

36.  $\cos 2z \equiv 1 - 2 \sin^2 z.$

37.  $\tan 2\alpha \equiv \frac{2 \tan \alpha}{1 - \tan^2 \alpha}.$

38.  $\tan(\alpha + \beta) \equiv \frac{\cot \beta + \cot \alpha}{\cot \alpha \cot \beta - 1}.$

39.  $(\sin x \cos y)^2 - (\cos x \sin y)^2 \equiv \sin^2 x - \sin^2 y.$

40.  $\sin(x + y) \sin(x - y) \equiv \sin^2 x - \sin^2 y.$