[NAM11-05-09-21]

[Exercise 1.3]

1. Convert the following to logarithmic form.
   (a) $2^4 = 16$ (b) $3^{-2} = \frac{1}{9}$
   (c) $100 = 10^2$ (d) $a^3 = y$
   (e) $2^x = p$ (f) $x^4 = 2 - k$

2. Convert the following to index form.
   (a) $3 = \log_5 125$ (b) $-2 = \log_2\left(\frac{1}{4}\right)$
   (c) $\log_4 64 = 3$ (d) $\log_x 3 = 4$
   (e) $\log_3 y = n$ (f) $p + 1 = \log_2 (4y)$

3. Check whether the logarithm $\log_x (5 - 2x)$ is defined when:
   (a) $x = 2$ (b) $x = 0.5$ (c) $x = 3$
   (d) $x = 2.5$ (e) $x = 1$ (f) $x = \sqrt{2}$

4. Solve the following equations.
   (a) $\log_2 x = 3$ (b) $\log_x 9 = 2$
   (c) $x = \log_4 8$ (d) $\log_3 (x - 2) = 1$
   (e) $\log_2 (2x + 1) = -3$ (f) $\log_9 \sqrt{27} = x$
   (g) $\log_x (6x - 8) = 2$ (h) $\log_x 8 = \frac{3}{2}$

5. Evaluate.
   (a) $\log_4 4 - 3 \log_2 2$ (b) $\log_2 1 + 2 \log_5 5$
   (c) $(3 - \log_3 3)^3$ (d) $\left(\frac{3 \log_x x + 2}{4 - 2 \log_5 1}\right)^2$
   (e) $\log_2 (6 - 5 \log_7 7)$

6. Given that $\log_4 x = 2$ and $\log_2 y = 3$, evaluate $\frac{x}{y}$.

7. Given that $\log_3 x = a$ and $\log_{\sqrt{3}} y = b$, express xy^2 as a power of 3.

8. Solve the following simultaneous equations.
   (a) $\log_x 16 = 4$ and $\log_2 y = x$ (b) $\log_y x = 2$ and $xy = 8$

*9. Given that $\log_4 y = a$ and $\log_8 (2y) = b$, show that $2a = 3b - 1$.

10. If $\log_2 (\log_3 x) = \log_5 5$, find x.

[Exercise 1.4]

1. Evaluate the following logarithms.

(a) $\log_2 4$ (b) $\lg\left(\frac{1}{10}\right)$ (c) $\log_2 8$

(d) $\log_3 27$ (e) $\log_2\left(\frac{1}{4}\right)$ (f) $\log_9 \sqrt{3}$

2. Evaluate the following expressions.

(a) $\log_8 2 + \log_8 4$ (b) $\log_3 36 - \log_3 12$

(c) $\log_2 60 - \log_2 15$ (d) $\log_3 4 + \log_3 2 - \log_3 72$

(e) $\log_6 54 - 2\log_6 3$ (f) $\log_5 4 + 2\log_5 3 - 2\log_5 6$

3. Simplify and express each of the following as a single logarithm.

(a) $\log_a 8 - 2\log_a 4$. (b) $2\log_x 5 - 3\log_x 2 + \log_x 4$

(c) $\lg\left(\frac{8}{75}\right) - 2\lg\left(\frac{3}{5}\right) + 4\lg\left(\frac{3}{2}\right)$

(d) $2\lg(x+2) + \lg(x+1) - \lg(x^2+3x+2)$

4. Evaluate the following:

(a) $\log_a \sqrt{a}$ (b) $\log_a \frac{1}{a^3}$

(c) $\dfrac{\log_a 9}{2\log_a 27}$ (d) $\log_{\sqrt{a}} a^2$

5. Given that $\log_a 3 = 0.477$ and $\log_a 5 = 0.699$, evaluate the following:

(a) $\log_a 15$ (b) $\log_a 3\sqrt{5}$ (c) $\log_a 0.6$

(d) $\dfrac{\log_a 25}{\log_a 3a}$ (e) $\log_a (5a^2)$ (f) $\log_a\left(\frac{9}{5a}\right)$

6. Given that $\log_4 3 = a$ and $\log_4 5 = b$, express the following in terms of a and b.

(a) $\log_4 45$ (b) $\log_4 20$ (c) $\log_4 75$

(d) $\log_4 (0.6)$ (e) $\log_4 (0.75)$ (f) $\log_4 (1.8)$

7. Given that $\lg x = p$ and $\lg y = q$, express the following in terms of p and q.

(a) $\lg(xy^2)$ (b) $\lg\left(\frac{10x}{y}\right)$ (c) $\lg\sqrt{10x^3y}$

(d) $\lg\left(\frac{100\sqrt{x}}{y^2}\right)$ (e) xy (f) $\lg(y^x)$

8. Find y in terms of x when

(a) $\lg y = 1 + 3\lg x$, (b) $\lg(y+1) = 2 - \frac{1}{2}\lg x$,

(c) $2\log_3 y - 4 = 3\log_3(x+2)$, (d) $3 + \log_2(x+y) = \log_2(x-2y)$.

9. If $\log_2(y+1) = 2\log_2 x + c$ and $y = 3$ when $x = 2$, find y in terms of x.

10. Express each of the following as a single logarithm.

(a) $2 + \log_3 5$ (b) $3 - 2\lg 5$ (c) $3\log_a 2 - 4 + \log_a a^3$

[Exercise 1.5]

For questions 1 to 10, solve for x.

1. $\log_2 (x - 1) = \log_2 (4x - 7)$

2. $\log_3 (x + 2) + \log_3 (x - 2) = \log_3 (2x - 1)$

3. $\lg 18 + \lg\left(\frac{1}{3}x\right) - \lg (x + 1) = 0$

4. $\log_3 x + \log_3 (x + 2) = 1$

5. $2 \lg 5 - \lg (x + 2) = 1 - \lg (2x - 1)$

6. $\log_2 (x - 2) + \log_2 (8 - x) - \log_2 (x - 5) = 3$

7. $\log_2 (x - 1)^2 = 2 + \log_2 (x + 2)$

8. $\log_3 (x + 2) + \log_3 (10 - x) - 3 = 0$

9. $3 \log_x 2 + \log_x 18 = 2$

10. $\log_p 2 + \log_p (x - 1) = 0$

11. Evaluate.

(a) $\log_3 5 \, . \log_5 27$

(b) $\dfrac{\log_5 4 \, . \log_2 10}{\log_{25}\sqrt{10}}$

12. If a, b and c are positive numbers other than 1, show that

$$\log_b a \, . \log_c b \, . \log_a c = 1.$$

13. By using the substitution $y = \log_3 x$ or otherwise, solve the equation

(a) $\log_3 x + 2 = 3 \log_x 3$,

(b) $\log_3 x^3 = (\log_3 x)^3$.

14. Solve the following equations.

(a) $\log_3 x = 9 \log_x 3$

(b) $4 \log_4 x - 9 \log_x 4 = 0$

(c) $\log_2 x = \log_4 (x + 6)$

(d) $\log_5 (5 - 4x) = \log_{\sqrt{5}} (2 - x)$

***15.** If $2 \log_a x = 1 + \log_a (7x - 10a)$, find x in terms of a.

***16.** Find x for which $27 \times 3^{\lg x} = 9^{1 + \lg (x - 20)}$.

ANSWERS

CHAPTER 1

Exercise 1.1 (p. 2)

1. (a) 1 (b) 4 (c) 144 (d) $\frac{1}{2}$ (e) 5 (f) $\frac{1}{3}$
2. (a) 54 (b) 4
3. (a) $8y$ (b) $\frac{1}{2}y^2$ (c) $\frac{12}{y}$ (d) $\frac{32}{y^3}$ (e) $\frac{1}{2}y^2$ (f) $y^3 - \frac{1}{y^2}$
4. (a) $18y^2$ (b) $\frac{1}{6}yz$ (c) $\frac{z^2}{y}$

Exercise 1.2 (p. 5)

1. (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) -2 (d) $-\frac{5}{2}$ (e) $\frac{1}{2}$ (f) 2
   (g) 2, -2 (h) 2 (i) $\frac{2}{3}$ (j) $\frac{5}{4}$ (k) 2 (l) -6, 2
   (m) -2, 4 (n) -2, 1
2. $a = 3$, $n = 2$
3. (a) $x = -\frac{4}{9}$, $y = \frac{1}{9}$ (b) $x = 2$, $y = 1$ (c) $x = 1$, $y = -1$
4. $m = 4$, $n = 3$
5. 3
6. (a) 0, 1 (b) 0, 2
7. (a) 0, 1 (b) 1, 3 (c) $-\frac{1}{2}$, $\frac{1}{2}$ (d) 0 (e) -2, 0 (f) 3
9. (a) 2 (b) ± 2 (c) 6 (d) 3 (e) 2 (f) 2
10. $r = 3$, $k = \frac{2}{3}$
11. $x = 1$, $y = -1$ or $x = 2$, $y = 1$
12. 4

Exercise 1.3 (p. 9)

1. (a) $4 = \log_2 16$ (b) $-2 = \log_3\left(\frac{1}{9}\right)$ (c) $2 = \log_{10} 100$
   (d) $3 = \log_a y$ (e) $x = \log_2 p$ (f) $4 = \log_x (2 - k)$
2. (a) $5^3 = 125$ (b) $2^{-2} = \frac{1}{4}$ (c) $4^3 = 64$
   (d) $x^4 = 3$ (e) $3^n = y$ (f) $2^{p+1} = 4y$
3. (c), (d) and (e) are not defined
4. (a) 8 (b) 3 (c) $\frac{3}{2}$ (d) 5
   (e) $-\frac{7}{16}$ (f) $\frac{3}{4}$ (g) 2, 4 (h) 4
5. (a) -2 (b) 2 (c) 8 (d) $\frac{25}{16}$ (e) 0
6. 2
7. 3^{a+b}
8. (a) $x = 2$, $y = 4$ (b) $x = 4$, $y = 2$
10. 9

Exercise 1.4 *(p. 14)*

1. (a) 2 (b) -1 (c) 3 (d) 3 (e) -2 (f) $\frac{1}{4}$

2. (a) 1 (b) 1 (c) 2 (d) -2 (e) 1 (f) 0

3. (a) $-\log_a 2$ (b) $\log_x\left(\frac{25}{2}\right)$ (c) $\lg\left(\frac{3}{2}\right)$ (d) $\lg(x+2)$

4. (a) $\frac{1}{2}$ (b) -3 (c) $\frac{1}{3}$ (d) 4

5. (a) 1.176 (b) 0.826 5 (c) -0.222 (d) 0.947 (e) 2.699 (f) -0.745

6. (a) $2a+b$ (b) $1+b$ (c) $a+2b$ (d) $a-b$ (e) $a-1$ (f) $2a-b$

7. (a) $p+2q$ (b) $1+p-q$ (c) $\frac{1}{2}(1+3p+q)$

(d) $2+\frac{1}{2}p-2q$ (e) 10^{p+q} (f) $q(10^p)$

8. (a) $y=10x^3$ (b) $y=\frac{100}{\sqrt{x}}-1$ (c) $y=9(x+2)^{\frac{3}{2}}$ (d) $y=-0.7x$

9. $y=x^2-1$

10. (a) $\log_3 45$ (b) $\lg 40$ (c) $\log_a\left(\frac{8}{a}\right)$

Exercise 1.5 *(p. 18)*

1. 2 **2.** 3 **3.** $\frac{1}{5}$ **4.** 1 **5.** $\frac{9}{8}$

6. 6 **7.** -1, 7 **8.** 1, 7 **9.** 12 **10.** $1\frac{1}{2}$

11. (a) 3 (b) 8 **13.** (a) 3, $\frac{1}{27}$ (b) 1, $3^{\sqrt{3}}$, $3^{-\sqrt{3}}$

14. (a) 27, $\frac{1}{27}$ (b) 8, $\frac{1}{8}$ (c) 3 (d) -1, 1

15. $2a$, $5a$ **16.** 40

Exercise 1.6 *(p. 20)*

1. (a) 0.954 (b) 1.39 (c) 6.80 (d) 5.14 (e) 2.37 (f) 1.13
(g) 1.65 (h) 0.291 (i) 4.32 (j) -0.168 (k) 3.32 (l) 3.32

2. (a) 1.21 (b) 2.18 (c) 1.20 (d) -3.17 (e) 0.639 (f) 2.08

3. (a) $y=\lg(x+1)$ (b) $y=1-\ln 3x$ (c) $y=e^x-1$
(d) $y=10^{\frac{1}{2}x-1}$ (e) $y=\frac{1}{2}\ln(x-4)$ (f) $y=e^{4x}-x$

4. (a) 4.07 (b) 5.65, 0.177 (c) 1.35 (d) 3.33×10^8 (e) 18.9
(f) 3.69 (g) 0.349 (h) 18.7 (i) 72.5

5. 5; 4; 7; 3 **6.** (a) 5.550 (b) $-1\,829$ (c) 2.484

Exercise 1.7 *(p. 22)*

1. 1.37 **2.** 6.48 **3.** 0.834 **4.** 0.282 **5.** 1.95 **6.** 0.880

7. 0.829 **8.** 1.21 **9.** 1.26 **10.** 2.58 **11.** 1.94 **12.** -0.713

13. 0.811 **14.** 2.58 **15.** ±2.40 **16.** 1.86

17. (a) 0.633 (b) 0.112 (c) -1.16 (d) -1.70

18. (a) 0.693 (b) 1.10, 1.39 (c) 0, 0.693

19. (a) $a=2$, $b=-\frac{1}{3}$ (b) $a=1$, $b=0.9$ (c) $a=-1.5$, $b=1.7$

20. (a) 9.11 (b) 4.38 **21.** $x=3$, $y=5$

22. (a) 1.26 (b) -1.39, 2.20