Exercises [A-1]

1. (a) Show that $(x-4)(x^2+x+2)+3$ is an alternative form of x^3-3x^2-2x-5.
   (b) What is the remainder when x^3-3x^2-2x-5 is divided by $x-4$?
   (c) What is the value of x^3-3x^2-2x-5 when $x=4$?
2. (a) If $f(x)=6x^3-28x^2+19x+3$, show that
$$(3x-2)(2x^2-8x+1)+5$$
   is an alternative form of $f(x)$.
   (b) What is the remainder when $f(x)$ is divided by $3x-2$?
   (c) Find $f(\frac{2}{3})$.
3. Find the quotient and remainder when $3x^3-4x^2-3x-2$ is divided by (a) $x-2$, (b) $x+1$.
4. Find the quotient and remainder when $2x^4-5x^2+2x-3$ is divided by (a) $x-1$, (b) $x+2$.
5. Find the quotient and remainder when $2x^3-3x^2-8x+4$ is divided by (a) $x-3$, (b) $2x+1$.
6. Show that $x+2$ is a factor of x^5+32.
7. Divide $4x^4-5x^2-8x-10$ by $2x-3$.
8. Find the value of k for which $x-3$ is a factor of $2x^3+4x^2-5x+k$.
9. Solve $x^3+2x-12=0$. Find the product of the three roots.
10. Find the value of k for which $x-1$ is a factor of x^4-4x+k, and using this value of k factor the polynomial completely.
11. Solve $2x^4+2x^3=(x+1)^2$.
12. Show that if a, b, c, d are all positive, then $ax^3-bx^2+cx-d=0$ cannot have a negative root.
13. Solve $2x^3-9x^2+14x-5=0$. Find the sum of the roots and the product of the roots.
14. Show that $x^5-3x^2=20$ is satisfied by $x=2$, and that the equation has no other positive root.
15. Solve $3x^3-x^2+2x-8=0$.
16. Show that $x^3=9x+3$ has no rational root but that it has a real root between 3 and 4.
17. Does $x^3+x=20$ have (a) a negative root, (b) a rational root? Show that the equation has a root between 2 and 3, and determine whether the root is greater or less than 2.5.

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1. b. 3
   c. 3
2. b. 5
   c. 5
3. a. $3x^2 + 2x + 1$; 0
   b. $3x^2 - 7x + 4$; -6
4. a. $2x^3 + 2x^2 - 3x - 1$; -4
   b. $2x^3 - 4x^2 + 3x - 4$; 5
5. a. $2x^2 + 3x + 1$; 7
   b. $x^2 - 2x - 3$; 7
7. $2x^3 + 3x^2 + 2x - 1$; R = -13
8. k = -75
9. $\{2, -1 + i\sqrt{5}, -1 - i\sqrt{5}\}$; 12
10. k = 3; $(x - 1)(x - 1)(x^2 + 2x + 3)$
11. $\{-1, 1, -\frac{1}{2} + \frac{1}{2}i, -\frac{1}{2} - \frac{1}{2}i\}$
13. $\{\frac{1}{2}, 2 + i, 2 - i\}$; $\frac{9}{2}$; $\frac{5}{2}$
15. $\{\frac{4}{3}, -\frac{1}{2} + \frac{1}{2}i\sqrt{7}, -\frac{1}{2} - \frac{1}{2}i\sqrt{7}\}$