

6 The fact that $i^2 = -1$ can be used to find higher powers of i , as shown in the following examples.

$$\begin{aligned} i^3 &= i \cdot i^2 = i(-1) = -i & i^6 &= i^2 \cdot i^4 = (-1) \cdot 1 = -1 \\ i^4 &= i^2 \cdot i^2 = (-1)(-1) = 1 & i^7 &= i^3 \cdot i^4 = (-i) \cdot 1 = -i \\ i^5 &= i \cdot i^4 = i \cdot 1 = i & i^8 &= i^4 \cdot i^4 = 1 \cdot 1 = 1 \end{aligned}$$

A few powers of i are listed here.

Powers of i	$i^1 = i$	$i^5 = i$	$i^9 = i$	$i^{13} = i$
	$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$	$i^{14} = -1$
	$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$	$i^{15} = -i$
	$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$	$i^{16} = 1$

As these examples show, the powers of i rotate through the four numbers i , -1 , $-i$, and 1 . This fact is used to find any power of i , as in the next example.

Example 8 Find each power of i .

(a) $i^{12} = (i^4)^3 = 1^3 = 1$

(b) $i^{39} = i^{36} \cdot i^3 = (i^4)^9 \cdot i^3 = 1^9 \cdot (-i) = -i$

(c) $i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$

(d) $i^{-1} = \frac{1}{i}$

Simplify this quotient by multiplying the numerator and the denominator by the conjugate of i , which is $-i$.

$$\frac{1}{i} = \frac{1(-i)}{i(-i)} = \frac{-i}{-i^2} = \frac{-i}{-(-1)} = \frac{-i}{1} = -i \quad \oplus$$

5.7 Exercises

Simplify. See Examples 1-3.

- | | | |
|----------------------------------|------------------------------------|---------------------------------------|
| 1. $\sqrt{-144}$ | 2. $\sqrt{-196}$ | 3. $-\sqrt{-225}$ |
| 4. $-\sqrt{-400}$ | 5. $\sqrt{-3}$ | 6. $\sqrt{-19}$ |
| 7. $\sqrt{-75}$ | 8. $\sqrt{-125}$ | 9. $4\sqrt{-12} - 3\sqrt{-3}$ |
| 10. $6\sqrt{-18} + 2\sqrt{-32}$ | 11. $2\sqrt{-45} + 4\sqrt{-80}$ | 12. $-3\sqrt{-28} + 2\sqrt{-63}$ |
| 13. $\sqrt{-5} \cdot \sqrt{-5}$ | 14. $\sqrt{-3} \cdot \sqrt{-3}$ | 15. $\sqrt{-9} \cdot \sqrt{-36}$ |
| 16. $\sqrt{-4} \cdot \sqrt{-81}$ | 17. $\sqrt{-16} \cdot \sqrt{-100}$ | 18. $\sqrt{-81} \cdot \sqrt{-121}$ |
| 19. $\sqrt{-27} \cdot \sqrt{-6}$ | 20. $\sqrt{-75} \cdot \sqrt{-20}$ | 21. $\frac{\sqrt{-200}}{\sqrt{-100}}$ |

22. $\frac{\sqrt{-50}}{\sqrt{-2}}$

23. $\frac{\sqrt{-54}}{\sqrt{6}}$

24. $\frac{\sqrt{-90}}{\sqrt{10}}$

25. $\frac{\sqrt{-288}}{\sqrt{-8}}$

26. $\frac{\sqrt{-300}}{\sqrt{-75}}$

27. $\frac{\sqrt{-48} \cdot \sqrt{-3}}{\sqrt{-2}}$

Add or subtract as indicated. Give answers in standard form. See Examples 4 and 5.

28. $(6 + 2i) + (4 + 3i)$

29. $(1 + i) + (2 - i)$

30. $(3 - 2i) + (-4 + 5i)$

31. $(7 + 15i) + (-11 - 14i)$

32. $(8 + 4i) + (-8 - 4i)$

33. $(-3 - 2i) + (3 + 2i)$

34. $(5 - i) + (-5 + i)$

35. $(-2 + 6i) + (2 - 6i)$

36. $(6 - 3i) - (-2 - 2i)$

37. $(5 - 8i) - (1 + 7i)$

38. $(5 + 5i) - (-2 - 3i)$

39. $(-1 - i) - (-3 - 4i)$

40. $(-3 - 4i) - (-1 - i)$

41. $(-2 - 3i) - (5 + 5i)$

42. $(6 + i) + (3 - 2i) + (2 + 5i)$

43. $(-8 + 3i) + (7 - 2i) + (8 + i)$

44. $(-4 + 11i) + (-2 - 4i) + (-3i)$

45. $(4 + i) + (1 + 5i) + (3 + 2i)$

46. $[(7 + 3i) - (4 - 2i)] + (3 - i)$

47. $[(3 + 2i) + (-4 - i)] - (2 + 5i)$

Multiply. Give answers in standard form. See Example 6.

48. $(8i)(2i)$

49. $(2i)(5i)$

50. $(4i)(7i)$

51. $(-3i)(-2i)$

52. $(-6i)(-5i)$

53. $2i(4 - 3i)$

54. $6i(1 - i)$

55. $i(5 + 2i)$

56. $-2i(3 + 7i)$

57. $(3 + 2i)(-1 + 2i)$

58. $(7 + 3i)(-5 + i)$

59. $(4 + 3i)(6 + 5i)$

60. $(-1 + 3i)(1 + 2i)$

61. $(3 + 5i)(-1 + 9i)$

62. $(2 + 6i)(1 + 3i)$

63. $(2 + 2i)^2$

64. $(-1 + 3i)^2$

65. $(-5 + 2i)^2$

66. $(6 + i)^2$

67. $(5 + 6i)(5 - 6i)$

68. $(2 + 3i)(2 - 3i)$

69. $(4 + 2i)(4 - 2i)$

70. $(1 + i)(1 - i)$

71. $(1 - 4i)(1 + 4i)$

72. $(5 - 2i)(5 + 2i)$

73. $(8 - 9i)(8 + 9i)$

74. $(3 + 7i)(3 - 7i)$

Find the quotients. Give answers in standard form. See Example 7.

75. $\frac{2}{1 + i}$

76. $\frac{-5}{2 - i}$

77. $\frac{i}{2 + i}$

78. $\frac{-i}{3 - 2i}$

79. $\frac{2 - 2i}{1 + i}$

80. $\frac{4i}{1 - i}$

81. $\frac{2 - 3i}{2 + 3i}$

82. $\frac{1 - i}{1 + i}$

83. $\frac{-1 + 5i}{3 + 2i}$

84. $\frac{3 + 4i}{-1 + 2i}$

85. $\frac{2 + 6i}{2 + i}$

86. $\frac{4 - 3i}{1 + 2i}$

87. $\frac{2}{5i}$

88. $\frac{-9}{2i}$

89. $\frac{-6}{5i}$

90. $\frac{10}{7i}$

Find each of the following powers of i . See Example 8.

91. i^6

92. i^9

93. i^{17}

94. i^{98}

95. i^{-3}

96. i^{-5}

97. i^{-10}

98. i^{-40}

Perform the indicated operations. Give answers in standard form.

99. $\frac{3}{2 - i} + \frac{5}{1 + i}$

100. $\frac{2}{3 + 4i} + \frac{4}{1 - i}$

Section 5.7 (page 242)

1. $12i$ 3. $-15i$ 5. $i\sqrt{3}$ 7. $5i\sqrt{3}$ 9. $5i\sqrt{3}$ 11. $22i\sqrt{5}$ 13. -5
 15. -18 17. -40 19. $-9\sqrt{2}$ 21. $\sqrt{2}$ 23. $3i$ 25. 6 27. $6i\sqrt{2}$
 29. $3 + 0i$ (written in the form $a + bi$) 31. $-4 + i$ 33. $0 + 0i$ (Note: $0 + 0i = 0$; however, $0 + 0i$ is in the form $a + bi$.) 35. $0 + 0i$ 37. $4 - 15i$ 39. $2 + 3i$ 41. $-7 - 8i$
 43. $7 + 2i$ 45. $8 + 8i$ 47. $-3 - 4i$ 49. $-10 + 0i$ 51. $-6 + 0i$ 53. $6 + 8i$
 55. $-2 + 5i$ 57. $-7 + 4i$ 59. $9 + 38i$ 61. $-48 + 22i$ 63. $0 + 8i$
 65. $21 - 20i$ 67. $61 + 0i$ 69. $20 + 0i$ 71. $17 + 0i$ 73. $145 + 0i$ 75. $1 - i$
 77. $\frac{1}{5} + \frac{2}{5}i$ 79. $0 - 2i$ 81. $-\frac{5}{13} - \frac{12}{13}i$ 83. $\frac{7}{13} + \frac{17}{13}i$ 85. $2 + 2i$
 87. $0 - \frac{2}{5}i$ 89. $0 + \frac{6}{5}i$ 91. -1 93. i 95. i 97. -1 99. $\frac{37}{10} - \frac{19}{10}i$
 101. $-\frac{294}{221} + \frac{14}{221}i$ 103. $-\frac{13}{10} + \frac{11}{10}i$ 105. $\frac{1}{2} + \frac{1}{2}i$ 109. $4, -4$ 111. $\sqrt{7}, -\sqrt{7}$
 113. $2\sqrt{3}, -2\sqrt{3}$ 115. $5i, -5i$

Chapter 5 Review Exercises (page 245)

1. 1000 3. $1/100$ 5. $1000/27$ 7. 5^2 or 25 9. $1/y^{1/2}$ 11. $r^{1/2} + r$ 13. 42
 15. -5 17. -2 19. $8|m^5|$ or $8m^4|m|$ 21. $3^8\sqrt{3}$ 23. $-y^3\sqrt[3]{y^2}$ 25. $\sqrt{78}$
 27. $\sqrt[3]{30}$ 29. $5\sqrt{5}$ 31. $10y^3\sqrt{y}$ 33. $7/9$ 35. $m^5/3$ 37. $\sqrt{5}$
 39. $2\sqrt{7}i$ 41. $\sqrt{30}/5$ 43. $5\sqrt{2}/4$ 45. $-\sqrt[3]{45}/5$ 47. $3m^3\sqrt[4]{p/p^2}$
 49. $-11\sqrt{2}$ 51. $23\sqrt{5}$ 53. $-8\sqrt[4]{2}$ 55. $1 - \sqrt{3}$ 57. $9 - 7\sqrt{2}$ 59. 29
 61. $-5(\sqrt{6} + \sqrt{3})/3$ 63. $(15r + 11\sqrt{15rs} + 10s)/(3r - 20s)$ 65. $\{6\}$ 67. $\{25\}$
 69. $\{3\}$ 71. $\{6\}$ 73. $5i$ 75. $10i\sqrt{2}$ 77. $-10 - 2i$ 79. $-8 + 4i$
 81. $3 + 0i$ 83. $42 - 32i$ 85. $-i$ 87. $-i$ 89. $7\sqrt{3}y$ 91. -4
 93. $26m\sqrt{6m}$ 95. $13|a|b^2$ 97. 6 99. $3\sqrt{7py}/y$ 101. 128 103. $7^2\sqrt[4]{7}$ or $49\sqrt[4]{7}$
 105. $86 + 8\sqrt{55}$ 107. $1 - i$ 109. $-5i$

Chapter 5 Test (page 247)

1. 81 2. 27 3. $1/3$ 4. $a^{11/4}$ 5. $r + r^{-1}$ or $r + 1/r$ 6. -10 7. $10y^7$
 8. $-2mp^3$ 9. Not a real number 10. $a^3\sqrt[3]{a^2}$ 11. $6x^2\sqrt{3x}$ 12. $2ab^2\sqrt[4]{ab^3}$
 13. $3\sqrt[3]{2}$ 14. $8\sqrt{5}$ 15. $12 - 2\sqrt{15}$ 16. $9\sqrt{5} - 8$ 17. $-9\sqrt{10}/20$
 18. $3\sqrt[3]{2}/2$ 19. $-4(\sqrt{7} + \sqrt{5})$ 20. $\{9\}$ 21. $\{-1\}$ 22. $\{-4\}$ 23. $-14 + 21i$
 24. $-2 + 16i$ 25. $\frac{1}{10} + \frac{7}{10}i$

CHAPTER 6

Section 6.1 (page 253)

1. $\{-7, 7\}$ 3. $\{-\sqrt{13}, \sqrt{13}\}$ 5. $\{-2\sqrt{3}, 2\sqrt{3}\}$ 7. $\{0, 4\}$ 9. $\{-9, -3\}$
 11. $\left\{\frac{1 + \sqrt{3}}{3}, \frac{1 - \sqrt{3}}{3}\right\}$ 13. $\left\{\frac{-1 + 2\sqrt{3}}{4}, \frac{-1 - 2\sqrt{3}}{4}\right\}$ 15. $\{-3i, 3i\}$
 17. $\{-6i\sqrt{2}, 6i\sqrt{2}\}$ 19. $\{5 + i, 5 - i\}$ 21. $\left\{\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right\}$ 23. 9
 25. 25 27. $25/4$ 29. $121/4$ 31. $\{-1, 3\}$ 33. $\{-3, 5\}$ 35. $\{-3, 5/2\}$
 37. $\{-3, 1/2\}$ 39. $\{1 + \sqrt{2}, 1 - \sqrt{2}\}$ 41. $\{-3 + \sqrt{2}, -3 - \sqrt{2}\}$ 43. $\{-1/3, 1\}$

