[07-02-21-12-Ph]

Answers

1. (*a*) $5mg$ (*b*) $3.5mg$ (*c*) $0.5mg$

3. (*a*) 60° (*b*) 6.26 m/s

5. x_0

7. (*a*) 0.989 m *Note:* The mass falls a distance $h = (4 + x) \sin 30°$, where x is the compression of the spring. If you neglect x in calculating the potential-energy loss, you get 0.885 for x. (*b*) 0.783 m (*c*) 1.54 m above spring (*d*) it will slide up and down a distance that decreases each time, and will eventually come to rest with the spring compressed about 8.6 cm

9. 8.83×10^{-2} J (*b*) $y = (0.9)^N H$ (*c*) 44

11. (*a*) $\frac{1}{2}mgR_E$ (*b*) $\sqrt{gR_E} = 7.91$ km/s

13. 6 m

15. 50.6 cm

17. (*a*) 5.10 m (*b*) 10.2 m

19. (*a*) $v = \sqrt{2g(2x - L)}$ (*b*) $a \geqslant g$

21. (*b*) when r decreases, E_k increases but U and E_{total} both decrease

23. (*a*) 463 m/s (*b*) 10.74 km/s (*c*) about 8%

Problems

1. A pendulum consists of a bob of mass m attached to a light rod of negligible mass and of length L. The other end of the rod is fixed at a frictionless pivot. The pendulum is released from rest when the bob is directly over the pivot. Find the force exerted by the rod on the bob when (*a*) the bob is at its lowest point, (*b*) the rod is at an angle of 30° below the horizontal, and (*c*) the rod is at an angle of 30° above the horizontal.

2. A pendulum bob is released from rest at an initial angle θ_0 measured from $\theta = 0$ at the lowest point. Prove that the tension in the string at the bottom is greater than the initial tension when the bob is at rest by the amount $3E_k/L$, where L is the length of the string and E_k is the maximum kinetic energy of the bob.

3. A child is swinging from a suspended rope 4.0 m long which will break when the tension becomes twice the weight of the child. (*a*) What is the greatest angle θ_0 the rope can make with the vertical during the swing if the rope is not to break? (*b*) What is the speed of the child when the rope breaks if the greatest angle is slightly greater than that found in (*a*)?

4. An elastic string 25 cm long obeys Hooke's law $F_x = -kx$, where x is the extension from equilibrium. When a 150-g object is suspended from the string, the string stretches 5 cm. If the object is attached to the end of the string and dropped from the point of support of the string, find the distance it falls before first coming to rest.

5. An elastic string has natural length a and spring constant k. When an object of mass m is hanging from it vertically, the string stretches by x_0. One end of the string is attached to the top of a frictionless plane inclined at 30° to the horizontal. With the string lying down the incline the object of mass m is attached to it and released with the string in its unstretched condition. How far does the object slide down the plane before coming to rest for the first time?

6. A small mass m slides without friction along the loop-the-loop track shown in Figure 7-16. The circular loop has radius R. The mass starts from rest at point P a distance h above the bottom of the loop. (a) What is the kinetic energy of m when it reaches the top of the loop? (b) What is its acceleration at the top of the loop assuming that it stays on the track? (c) What is the least value of h if m is to reach the top of the loop without leaving the track? (d) Assuming that h is greater than this least value, write an expression for the normal force exerted by the track on the mass.

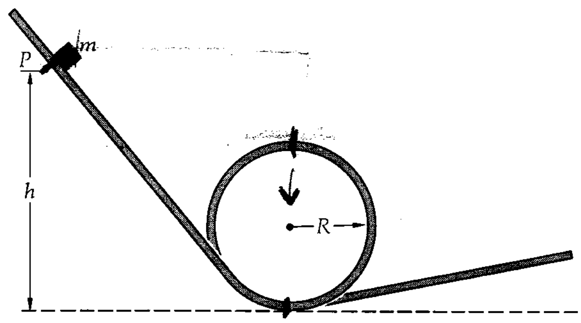

Figure 7-16
Problem 6.

7. A 2-kg mass is released on a frictionless incline 4 m from a spring of constant $k = 100$ N/m. The spring is fixed along the plane inclined at $\theta = 30°$ (Figure 7-17). (a) Find the maximum compression of the spring, assumed to be massless. (b) If the incline is not frictionless but the coefficient of friction between it and the mass is 0.2, find the maximum compression. (c) For the rough incline, how far up the incline will the mass travel after leaving the spring? (d) Describe the subsequent motion of the mass for the rough incline.

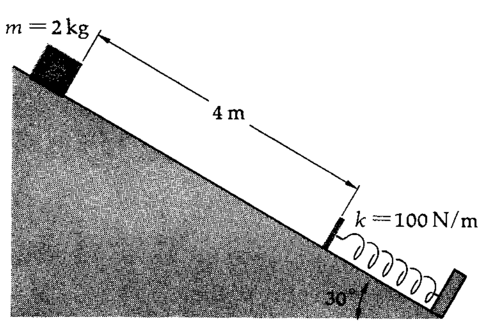

Figure 7-17
Problem 7.

8. For the arrangement in Figure 7-13 with general masses m_1 and m_2, (a) find an expression for the speed of the masses after m_2 has fallen a distance y. (b) Differentiate this expression to obtain an expression for the acceleration of the masses.

9. A Superball can bounce to 90 percent of its original height. (a) How much energy is lost after a 30-g ball is bounced once from an initial height of 3 m? (b) A ball dropped from an original height of H makes N bounces. Find a general expression for the maximum height of the ball after N bounces for a ball of mass m dropped from height H. (c) About how many bounces are required if the maximum height after the Nth bounce is 1 percent of the original height?

10. A particle of mass m is dropped from a height h which is not necessarily small compared with the radius of the earth R_E. Show that if air resistance is neglected, the speed of the particle when it reaches the surface of the earth is given by $v = \sqrt{2gh}\,\sqrt{R_E/(R_E + h)}$.

11. A hole is drilled from the surface of the earth to its center. The gravitational force on a particle of mass m which is inside the earth at a distance r from the center ($r \leq R_E$) has the magnitude mgr/R_E and points toward the center of the earth. (*a*) How much work is required to lift the particle from the center of the earth to the earth's surface? (*b*) If the particle is dropped from rest at the surface of the earth, what is its speed when it reaches the center of the earth?

12. Sketch the gravitational potential energy of a particle of mass m as a function of the distance r from the center of the earth using the result of part (*a*) of Problem 11 for $r \leq R_E$. Choose the potential to be zero at $r = \infty$. *Hint:* At $r = R_E$ both the potential energy $U(r)$ and its slope are continuous.

13. A skier starts from rest at height H above the center of a rounded hummock of radius 4.0 m (Figure 7-18). There is negligible friction. Find the maximum value of H for which the skier remains in contact with the snow at the peak of the hummock.

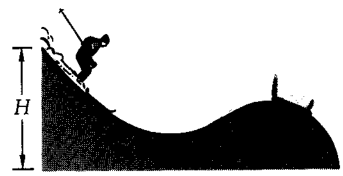

Figure 7-18
Problems 13 and 14.

14. A skier of mass 70 kg starts with a small initial speed from the top of the rounded hummock of Problem 13 (Figure 7-18). If friction can be neglected, find (*a*) his speed v as a function of the angle θ and (*b*) the angle θ at which he loses contact with the slope.

15. In Example 7-5 find the distance the block moves up the incline if the beginning of the incline is 40 cm from the point where the block leaves the spring (the natural length of the spring) and the coefficient of friction between the block and both the horizontal surface and the incline is $\mu_k = 0.20$.

16. Work Problem 8 if the coefficient of friction between block 1 and the horizontal surface is μ_k (see Figure 7-13).

17. A 15-g ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring can be compressed 5 cm. (*a*) How high can the ball be shot if the gun is aimed vertically? (*b*) What is the greatest possible horizontal range of the ball for this compression?

18. The rate of energy loss of a certain system at a given time is directly proportional to the total energy of the system at that time. (*a*) Write a differential equation relating the energy E to the rate of change of energy dE/dt expressing the above property. Let the proportionality constant be C. (*b*) Solve your equation to obtain a general relationship for the energy of the system E as a function of time t. (*c*) Find the constant C in your result when 10 percent of the energy is lost in 10 s. (*d*) How long does it take for half the energy of the system to be lost when C has the value found in part (*c*)?

19. In the problem of the pendulum string hitting a peg (Exercise 13) the pendulum is released from rest at $\theta = 90°$. (*a*) Find the speed of the bob in terms of g, L, and x when it is directly above the peg and moving in a circle of radius $L - x$. (*b*) What is the least value of its acceleration at that point if the string is not to go slack? (*c*) Show that the bob will not reach this point with the string still taut unless x is greater than or equal to $3L/5$.

20. A particle moves under the influence of a conservative force along the x axis with total energy E. The potential energy associated with the force is $U(x)$. (*a*) Show that the speed of the particle is given by $v = \sqrt{2/m}\,\sqrt{E - U}$. (*b*) Show that the distance dx traveled during the time interval dt is given by

$$\frac{dx}{\sqrt{E - U(x)}} = \sqrt{\frac{2}{m}}\,dt$$

(*c*) For a mass on a spring the potential energy is $U = \frac{1}{2}kx^2$, where k is the force constant, and the total energy is $E = \frac{1}{2}kA^2$, where A is the maximum displacement. Integrate your result in part (*b*) to find the total time taken for a mass on a spring to move from $x = -A$ to $x = +A$.

21. (*a*) Show that the total energy of a satellite in a circular orbit of radius r is given by $E = -GM_Em/2r$, where M_E is the mass of the earth and m the mass of the satellite, and that the speed v is given by $v = \sqrt{GM_E/r}$. (*b*) If you wish to decrease the radius of the circular orbit of a satellite, must you increase or decrease the kinetic energy of the satellite? How do the potential energy and total energy change if the radius is decreased? (*c*) Using the differential approximation $\Delta v \approx dv$ and $\Delta r \approx dr$, show that a small change in radius must be accompanied by a small change in speed given by

$$\frac{\Delta v}{v} = -\frac{1}{2}\frac{\Delta r}{r}$$

22. A space probe launched from the surface of the earth accelerates for a short distance Δr and then moves under the influence of gravity only. Since it is already a short distance Δr above the surface of the earth, its escape speed is slightly less than that at the earth's surface by amount Δv. (*a*) Find Δv in terms of Δr using the differential approximation $\Delta v \approx dv$ and $\Delta r \approx dr$. (*b*) Find the escape speed for $\Delta r = 300$ km.

23. In calculating the escape speed the rotation of the earth was neglected. The speed that must be given to a body relative to the ground will be less if the body is launched in the direction of the earth's rotation and more if it is launched away from it. Take the earth's rotation into account for a body launched on the equator in the direction of motion of the earth's surface (horizontally eastward). (*a*) What is the speed relative to the center of the earth of an object at rest on the equator? (*b*) Relative to the earth's surface, what speed must be given to a body for it to escape the earth? (*c*) By what percentage is the work required to accelerate the body reduced because of the earth's rotation?

24. The bob of a pendulum of length L is pulled aside so the string makes an angle θ_0 and released. In Example 7-3 energy conservation was used to obtain the speed at the bottom (Equation 7-7). In this problem you are to obtain this result using Newton's second law. (*a*) Show that the tangential component of Newton's second law gives $dv/dt = -g \sin \theta$, where v is the speed. (*b*) Show that v can be written

$$v = L\frac{d\theta}{dt}$$

where θ is the angle made by the string and the vertical. (*c*) Use your results from (*a*) and (*b*) and the chain rule

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

to obtain

$$v\,dv = -gL \sin \theta\, d\theta$$

(*d*) Integrate the left side of this equation from $v = 0$ to the final speed v and the right side from $\theta = \theta_0$ to $\theta = 0$ to obtain Equation 7-7.