[06-09-08-T12P]
Vector Summary

■ [11.2]

Unit vector

$\vec{u} = \dfrac{\vec{a}}{a}$

$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

■ [11.3] Dot Product

Dot Product

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{a} \cdot \vec{b} = ab \cos\theta$

$\vec{a} \cdot \vec{a} = a^2$

$\vec{a} \cdot \vec{b} = 0 \Longleftrightarrow \vec{a} \perp \vec{b}$

Direction Cosines

$\dfrac{\vec{a}}{a} = \dfrac{< a_1, a_2, a_3 >}{a} = < \cos\alpha, \cos\beta, \cos\gamma >$

Direction Cosines: α, β, γ

Scalar projection (component of $\vec{b}$ along $\vec{a}$)

$\text{comp}_a \vec{b} = \vec{b} \cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{a}$

Vector Projection of $\vec{b}$ onto $\vec{a}$

$\text{proj}_a \vec{b} = \vec{b} \cos\theta \left(\dfrac{\vec{a}}{a} \right) = \left(\dfrac{\vec{a} \cdot \vec{b}}{a} \right) \dfrac{\vec{a}}{a} = \left(\dfrac{\vec{a} \cdot \vec{b}}{a^2} \right) \vec{a}$

■ [11.4] Cross Product

Cross product

$$\vec{a}\times\vec{b} = \text{Det}\begin{pmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\hat{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\hat{k}$$

$\vec{a}\times\vec{b} = ab\sin\theta$, sense by RH – Rule

Direction of $\vec{\boldsymbol{a}}\times\vec{\boldsymbol{b}}$ is perpendicular to $\vec{\boldsymbol{a}}$ and to $\vec{\boldsymbol{b}}$.

Geometry

$\vec{a}$ and $\vec{b}$ are parallel $\Longrightarrow \vec{a}\times\vec{b} = 0$, i.e. $\theta = 0$ or $\theta = \pi$

$A_{\text{parallelogram}} = |\vec{a}\times\vec{b}|$

$V_{\text{parallelepipid}} = \vec{a}\cdot(\vec{b}\times\vec{c})$ ie. triple scalar product

■ [11.5] EQNS of lines and planes

■ Vector eqn of line

$\vec{r} = \vec{r_0} + t\,\vec{v}$

■ Parametric eqns of a line

If $\vec{v} = \langle a, b, c\rangle$ and $x_0 = \langle x_0, y_0, z_0\rangle$ and $r = \langle x, y, z\rangle$ then

$x = x_o + at \quad y = y_o + bt \quad z = z_o + ct$

and **a**, **b**, **c** are called "direction numbers"

■ Symmetric eqns of a line

■ $$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

■ Vector eqn of a plane

$\vec{n}\cdot(\vec{r} - \vec{r}_0) = \vec{0}$ or $\vec{n}\cdot\vec{r} = \vec{n}\cdot\vec{r}_0$

■ Scalar eqn of a plane

If $\vec{n} = \langle a, b, c \rangle$ then

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

or

$ax + by + cz = d$, (linear eqn in x and y)

Two planes are parallel if their normal vectors are parallel.
The angle between two intersecting planes equals the angle between their normal vectors.

Distance **D** from $\boldsymbol{P_1(x_1, y_1, z_1)}$ to the plane $\mathbf{ax + by + cz} + \boldsymbol{d} = \mathbf{0}$ is

$$D = \text{comp}_{\vec{n}} \vec{b} = \frac{|\vec{n} \cdot \vec{b}|}{n} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$