EXERCISES 6.3

1–8 ■ Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

1. $y = x^2,\ y = 0,\ x = 1,\ x = 2$
2. $y = 1/x,\ y = 0,\ x = 1,\ x = 10$
3. $y = e^{-x^2},\ y = 0,\ x = 0,\ x = 1$
4. $y = \sin(x^2),\ y = 0,\ x = 0,\ x = \sqrt{\pi}$
5. $y = x^2,\ y = 4,\ x = 0$
6. $y^2 = x,\ x = 2y$
7. $y = x^2 - x^3,\ y = 0$
8. $y = x^2 - 6x + 10,\ y = -x^2 + 6x - 6$

9–14 ■ Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

9. $x = \sqrt[4]{y},\ x = 0,\ y = 16$
10. $x = y^2,\ x = 0,\ y = 2,\ y = 5$
11. $y = x^2,\ y = 9$
12. $y^2 - 6y + x = 0,\ x = 0$
13. $y = \sqrt{x},\ y = 0,\ x + y = 2$
14. $y = x,\ x = 0,\ x + y = 2$

15–20 ■ Use the method of cylindrical shells as in Example 4 to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.

15. $y = \sqrt{x},\ y = 0,\ x = 1,\ x = 4$; about the y-axis
16. $y = x^2,\ y = 0,\ x = -2,\ x = -1$; about the y-axis
17. $y = x^2,\ y = 0,\ x = 1,\ x = 2$; about $x = 1$
18. $y = x^2,\ y = 0,\ x = 1,\ x = 2$; about $x = 4$
19. $y = \sqrt{x - 1},\ y = 0,\ x = 5$; about $y = 3$
20. $y = 4x - x^2,\ y = 8x - 2x^2$; about $x = -2$

21–26 ■ Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

21. $y = \sin x,\ y = 0,\ x = 2\pi,\ x = 3\pi$; about the y-axis
22. $y = 1/(1 + x^2),\ y = 0,\ x = 0,\ x = 3$; about the y-axis
23. $x = \cos y,\ x = 0,\ y = 0,\ y = \pi/4$; about the x-axis
24. $y = -x^2 + 7x - 10,\ y = x - 2$; about the x-axis
25. $y = x^4,\ y = \sin(\pi x/2)$; about $x = -1$
26. $x = 4 - y^2,\ x = 8 - 2y^2$; about $y = 5$

27–30 ■ Each integral represents the volume of a solid. Describe the solid.

27. $\displaystyle\int_0^{\pi/2} 2\pi x \cos x\, dx$
28. $\displaystyle\int_0^{9} 2\pi y^{3/2}\, dy$
29. $\displaystyle\int_0^{1} 2\pi(x^3 - x^7)\, dx$
30. $\displaystyle\int_0^{\pi} 2\pi(4 - x)\sin^4 x\, dx$

31–32 ■ Use a graph to estimate the x-coordinates of the points of intersection of the given curves. Then use this information to estimate the volume of the solid obtained by rotating about the y-axis the region enclosed by these curves.

31. $y = 0,\ y = x + x^2 - x^4$
32. $y = x^4,\ y = 3x - x^3$

33–38 ■ The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

33. $y = x^2 + x - 2,\ y = 0$; about the x-axis
34. $y = x^2 - 3x + 2,\ y = 0$; about the y-axis
35. $x = 1 - y^2,\ x = 0$; about the y-axis
36. $y = x\sqrt{1 + x^3},\ y = 0,\ x = 0,\ x = 2$; about the y-axis
37. $x^2 + (y - 1)^2 = 1$; about the y-axis
38. $x^2 + (y - 1)^2 = 1$; about the x-axis

39–41 ■ Use cylindrical shells to find the volume of each solid.

39. A sphere of radius r
40. The solid torus of Exercise 63 in Section 6.2

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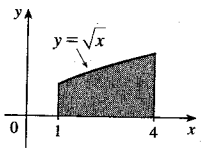
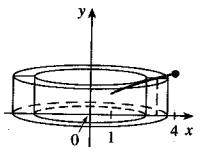
1. $15\pi/2$ **3.** $\pi(1 - 1/e)$ **5.** 8π **7.** $\pi/10$

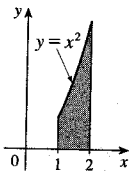
9. $4096\pi/9$ **11.** $1944\pi/5$ **13.** $5\pi/6$

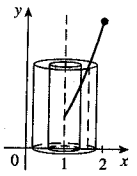
15. $124\pi/5$



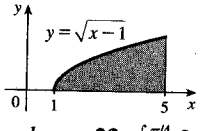


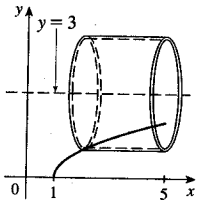
17. $17\pi/6$





19. 24π





21. $\int_{2\pi}^{3\pi} 2\pi x \sin x \, dx$ **23.** $\int_0^{\pi/4} 2\pi y \cos y \, dy$

25. $\int_0^1 2\pi(x + 1)[\sin(\pi x/2) - x^4] \, dx$

27. Solid obtained by rotating the region under $y = \cos x$ from 0 to $\pi/2$ about the y-axis

29. Solid obtained by rotating the region bounded by $y = x^2$ and $y = x^6$ about the y-axis

31. 0, 1.32; 4.05 **33.** $81\pi/10$ **35.** $16\pi/15$ **37.** $4\pi/3$

39. $\frac{4}{3}\pi r^3$ **41.** $\frac{1}{3}\pi r^2 h$ **45.** 1.142