EXERCISES 6.2

1–12 ■ Find the volume of the solid obtained by rotating the region bounded by the given curves about the given axis. Sketch the region, the solid, and a typical disk or "washer."

1. $y = x^2,\ x = 1,\ y = 0$; about the x-axis

2. $y^2 = x^3,\ x = 4,\ y = 0$; about the x-axis

3. $x + y = 1,\ x = 0,\ y = 0$; about the x-axis

4. $y = \sqrt{x - 1},\ x = 2,\ x = 5,\ y = 0$; about the x-axis

5. $y = x^2,\ y = 4,\ x = 0,\ x = 2$; about the y-axis

6. $x = y - y^2,\ x = 0$; about the y-axis

7. $y = x^2,\ y^2 = x$; about the x-axis

8. $y = x^2 + 1,\ y = 3 - x^2$; about the x-axis

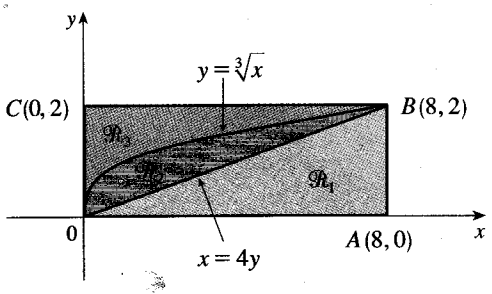
9. $y^2 = x,\ x = 2y$; about the y-axis

10. $y = 2x - x^2,\ y = 0,\ x = 0,\ x = 1$; about the y-axis

11. $y = x^4,\ y = 1$; about $y = 2$

12. $y = x,\ y = 0,\ x = 2,\ x = 4$; about $x = 1$

13–24 ■ Refer to the figure and find the volume generated by rotating the given region about the given line.



13. $\mathcal{R}_1$ about OA

14. $\mathcal{R}_1$ about OC

15. $\mathcal{R}_1$ about AB

16. $\mathcal{R}_1$ about BC

17. $\mathcal{R}_2$ about OA

18. $\mathcal{R}_2$ about OC

19. $\mathcal{R}_2$ about BC

20. $\mathcal{R}_2$ about AB

21. $\mathcal{R}_3$ about OA

22. $\mathcal{R}_3$ about OC

23. $\mathcal{R}_3$ about BC

24. $\mathcal{R}_3$ about AB

25–32 ■ Find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

25. $y = x^2 - 1,\ y = 0,\ x = 0,\ x = 2$

26. $y = -1/x,\ y = 0,\ x = 1,\ x = 3$

27. $y = e^x,\ y = 0,\ x = 0,\ x = 1$

28. $y = 1/\sqrt{x + 1},\ y = 0,\ x = 0,\ x = 1$

29. $y = \sec x,\ y = 1,\ x = -1,\ x = 1$

30. $y = \cos x,\ y = \sin x,\ x = 0,\ x = \pi/4$

31. $y = |x + 2|,\ y = 0,\ x = -3,\ x = 0$

32. $y = [\![x]\!],\ x = 1,\ x = 6,\ y = 0$

33–38 ■ Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the given line.

33. $y = \ln x,\ y = 1,\ x = 1$; about the x-axis

34. $y = \sqrt{x - 1},\ y = 0,\ x = 5$; about the y-axis

35. $x - y = 1,\ y = (x - 4)^2 + 1$; about $y = 7$

36. $y = \cos x,\ y = 0,\ x = 0,\ x = \pi/2$; about $y = 1$

37. $y = \cos x,\ y = 0,\ x = 0,\ x = \pi/2$; about $y = -1$

38. $2x + 3y = 6,\ (y - 1)^2 = 4 - x$; about $x = -5$

39–40 ■ Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the volume of the solid obtained by rotating about the x-axis the region bounded by these curves.

39. $y = x^2,\ y = \sqrt{x + 1}$

40. $y = x^4,\ y = 3x - x^3$

41–42 ■ Sketch and find the volume of the solid obtained by rotating the region under the graph of f about the x-axis.

41. $f(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x < 4 \\ 3 & \text{if } 4 \leq x \leq 5 \end{cases}$

42. $f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x < 1 \\ x^2 - 2x + 2 & \text{if } 1 \leq x \leq 2 \end{cases}$

43–48 ■ Each integral represents the volume of a solid. Describe the solid.

43. $\pi \int_0^{\pi/4} \tan^2 x\, dx$

44. $\pi \int_1^2 y^6\, dy$

45. $\pi \int_0^1 (y - y^2)\, dy$

46. $\pi \int_0^4 [16 - (x - 2)^4]\, dx$

47. $\pi \int_0^1 [(5 - 2x^2)^2 - (5 - 2x)^2]\, dx$

48. $\pi \int_{\pi/4}^{\pi/2} [(2 + \sin x)^2 - (2 + \cos x)^2]\, dx$

49–61 ■ Find the volume of the described solid S.

49. A right circular cone with height h and base radius r

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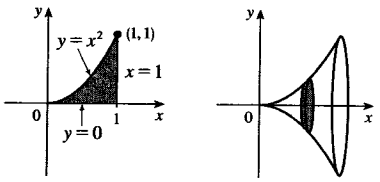
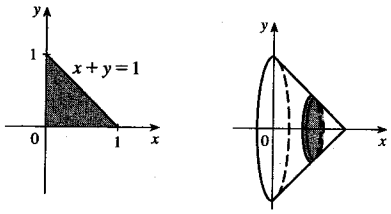
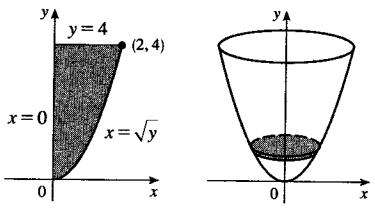
1. $\pi/5$



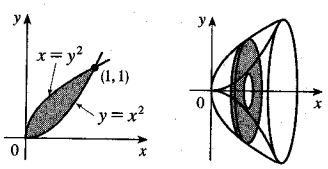
3. $\pi/3$



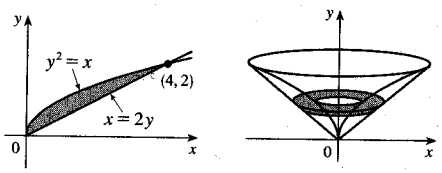
5. 8π



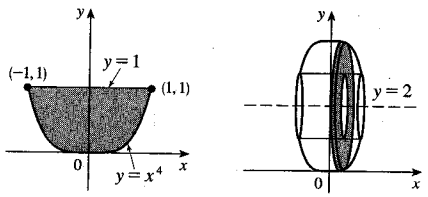
7. $3\pi/10$



9. $64\pi/15$



11. $208\pi/45$



13. $32\pi/3$ **15.** $128\pi/3$ **17.** $128\pi/15$ **19.** $112\pi/15$

21. $64\pi/5$ **23.** $16\pi/5$ **25.** $46\pi/15$ **27.** $\pi(e^2 - 1)/2$

29. $2\pi(\tan 1 - 1)$ **31.** 3π **33.** $\pi \int_1^e [1 - (\ln x)^2]\,dx$

35. $\pi \int_3^6 \{[6 - (x - 4)^2]^2 - (8 - x)^2\}\,dx$

37. $\pi \int_0^{\pi/2} [(1 + \cos x)^2 - 1]\,dx$ **39.** -0.72 1.22; 5.80