[06-12-19-TF12-B]

In each of the following problems (1 through 6), find the position s as a function of t from the given velocity $v = ds/dt$. Evaluate the constant of integration so as to have $s = s_0$ when $t = 0$.

1. $v = 3t^2$ **2.** $v = 2t + 1$

3. $v = (t + 1)^2$ **4.** $v = (t^2 + 1)^2$

5. $v = (t + 1)^{-2}$ **6.** $v = \sqrt{2gs}$ (g = constant)

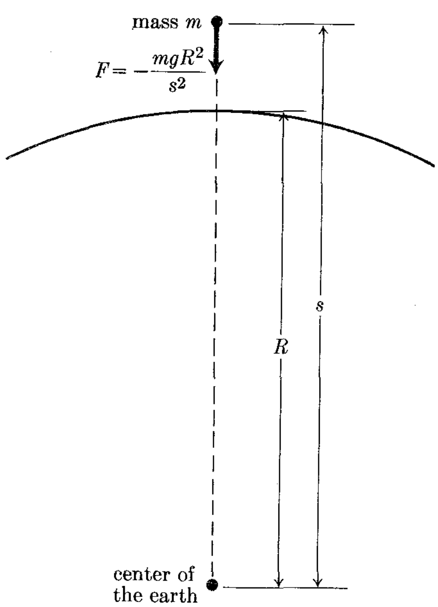
In each of the following problems (7 through 11), find the velocity v and position s as functions of t from the given acceleration $a = dv/dt$. Evaluate the constants of integration so as to have $v = v_0$ and $s = s_0$ when $t = 0$.

7. $a = g$ (constant) **8.** $a = t$

9. $a = \sqrt[3]{2t + 1}$ **10.** $a = (2t + 1)^{-3}$

11. $a = (t^2 + 1)^2$

12. The gravitational attraction exerted by the earth on a particle of mass m at distance s from the center is given by $F = -mgR^2s^{-2}$, where R is the radius of the earth and F is negative because the force acts in opposition to increasing s (Fig. 4–2). If a particle is projected vertically upward from the surface of the earth with initial velocity $v_0 = \sqrt{2gR}$, apply Newton's second law $F = ma$ with $a = v(dv/ds)$ to show that $v = v_0\sqrt{R/s}$ and that $s^{3/2} = R^{3/2}[1 + (3v_0 t/2R)]$.



4–2 A mass m that is s km from the earth's center.

REMARK. The initial velocity $v_0 = \sqrt{2gR}$ (approximately 11.2 kilometers per second) is known as the "velocity of escape," since the displacement s tends to infinity with increasing t provided the initial velocity is this large. Actually, a somewhat larger initial velocity is required for escape from the earth's gravitational attraction, due to the retardation effect of air resistance, which we have neglected here for the sake of simplicity.

Solve the following differential equations subject to the prescribed initial conditions.

13. $\frac{dy}{dx} = 9x^2 - 4x + 5$, $x = -1$, $y = 0$

14. $\frac{dy}{dx} = 4(x - 7)^3$, $x = 8$, $y = 10$

15. $\frac{dy}{dx} = x^{1/2} + x^{1/4}$, $x = 0$, $y = -2$

16. $\frac{dy}{dx} = \frac{x^2 + 1}{x^2}$, $x = 1$, $y = 1$

17. $\frac{dy}{dx} = x\sqrt{y}$, $x = 0$, $y = 1$

18. $\frac{dy}{dx} = 2xy^2$, $x = 1$, $y = 1$

19. $\frac{dy}{dx} = x\sqrt{1 + x^2}$, $x = 0$, $y = -3$

20. $\frac{dy}{dx} = \frac{4\sqrt{(1 + y^2)^3}}{y}$, $x = 0$, $y = 1$

Article 4–3, p. 184

1. $s = t^3 + s_0$ **2.** $s = t^2 + t + s_0$ **3.** $s = \frac{1}{3}(t + 1)^3 - \frac{1}{3} + s_0$ **4.** $s = \frac{1}{5}t^5 + \frac{2}{3}t^3 + t + s_0$ **5.** $s = -(t + 1)^{-1} + 1 + s_0$ **6.** $s =$ [illegible] $+ t\sqrt{2gs_0} + \frac{1}{2}gt^2$ **7.** $v = gt + v_0$; $s = \frac{1}{2}gt^2 + v_0 t + s_0$ **8.** $v = \frac{1}{2}t^2 + v_0$; $s = \frac{1}{6}t^3 + v_0 t + s_0$ **9.** $v = \frac{3}{8}(2t + 1)^{4/3} +$ [illegible] $s = \frac{9}{112}(2t + 1)^{7/3} + (v_0 - \frac{3}{8})t + s_0 - \frac{9}{112}$ **10.** $v = -\frac{1}{4}(2t + 1)^{-2} + v_0 + \frac{1}{4}$; $s = \frac{1}{8}(2t + 1)^{-1} + (v_0 + \frac{1}{4})t + s_0 - \frac{1}{8}$ **11.** [illegible] $=$ $\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + v_0$; $s = \frac{1}{30}t^6 + \frac{1}{6}t^4 + \frac{1}{2}t^2 + v_0 t + s_0$ **13.** $y = 3x^3 - 2x^2 + 5x + 10$ **14.** $y = (x - 7)^4 + 9$
15. $y = \frac{2}{3}x^{3/2} + \frac{4}{5}x^{5/4} - 2$ **16.** $y = x - (1/x) + 1$ **17.** $4\sqrt{y} = x^2 + 4$ **18.** $-y^{-1} = x^2 - 2$ **19.** $(3y + 10)^2 = (1 + x^2)^3$
20. $(1 + y^2)^{-(1/2)} = (1/\sqrt{2}) - 4x$ **21. a)** $R(x) = 250\sqrt{x} + (100/x) - 101$ **b)** \$1,035.03