

# [06-12-19-TF12-A]

Evaluate the following integrals.

1.  $\int \sin 3x \, dx$

2.  $\int \cos(2x + 4) \, dx$

3.  $\int x \sin(2x^2) \, dx$

4.  $\int (\cos \sqrt{x}) \frac{dx}{\sqrt{x}}$

5.  $\int \sin 2t \, dt$

6.  $\int \cos(3\theta - 1) \, d\theta$

7.  $\int 4 \cos 3y \, dy$

8.  $\int 2 \sin z \cos z \, dz$

9.  $\int \sin^2 x \cos x \, dx$

10.  $\int \cos^2 2y \sin 2y \, dy$

11.  $\int (1 - \sin^2 3t) \cos 3t \, dt$

12.  $\int \frac{\sin x \, dx}{\cos^2 x}$

13.  $\int \frac{\cos x \, dx}{\sin^2 x}$

14.  $\int \sqrt{2 + \sin 3t} \cos 3t \, dt$

15.  $\int \frac{\sin 2t \, dt}{\sqrt{2 - \cos 2t}}$

16.  $\int \sin^3 \frac{y}{2} \cos \frac{y}{2} \, dy$

17.  $\int \frac{\sin [(z-1)/3] \, dz}{\cos^2 [(z-1)/3]}$

18.  $\int \cos^2 \frac{2x}{3} \sin \frac{2x}{3} \, dx$

ii)  $\int 2 \sin x \cos x \, dx = \int -2 \cos x \, d(\cos x)$

$$= -\cos^2 x + C_2$$

19.  $\int (1 + \sin 2t)^{3/2} \cos 2t \, dt$

20.  $\int (3 \sin 2x + 4 \cos 3x) \, dx$

21.  $\int \sin t \cos t (\sin t + \cos t) \, dt$

22.  $\int (\sin x + x \cos x) \, dx$

Solve the following differential equations subject to the given initial conditions.

23.  $2y \frac{dy}{dx} = 5x - 3 \sin x, \quad x = 0, \quad y = 0$

24.  $\frac{dy}{dx} = \frac{\pi \cos \pi x}{\sqrt{y}}, \quad x = \frac{1}{2}, \quad y = 1$

25. We can treat the integral of  $2 \sin x \cos x \, dx$  in three different ways:

i)  $\int 2 \sin x \cos x \, dx = \int 2 \sin x \, d(\sin x)$   
 $= \sin^2 x + C_1$

iii)  $\int 2 \sin x \cos x \, dx = \int \sin 2x \, dx$   
 $= -\frac{1}{2} \cos 2x + C_3.$

Can all three integrations be correct? Explain.

## Article 4-4, pp. 187-188

1.  $-\frac{1}{3} \cos 3x + C$  2.  $\frac{1}{2} \sin(2x + 4) + C$  3.  $-\frac{1}{4} \cos(2x^2) + C$  4.  $2 \sin \sqrt{x} + C$  5.  $-\frac{1}{2} \cos 2t + C$  6.  $\frac{1}{3} \sin(3\theta - 1) + C$   
 7.  $\frac{4}{3} \sin 3y + C$  8.  $\sin^2 z + C$  9.  $\frac{1}{3} \sin^3 x + C$  10.  $-\frac{1}{8} \cos^3 2y + C$  11.  $\frac{1}{3} \sin 3t - \frac{1}{9} \sin^3 3t + C$  12.  $(1/\cos x) + C$   
 13.  $-(1/\sin x) + C$  14.  $\frac{2}{3}(2 + \sin 3t)^{3/2} + C$  15.  $\sqrt{2 - \cos 2t} + C$  16.  $\frac{1}{2} \sin^4(y/2) + C$  17.  $3 \sec [(z-1)/3] + C$   
 18.  $-\frac{1}{2} \cos^3(2x/3) + C$  19.  $\frac{1}{2}(1 + \sin 2t)^{5/2} + C$  20.  $-\frac{3}{2} \cos 2x + \frac{4}{3} \sin 3x + C$  21.  $\frac{1}{2}(\sin^3 t - \cos^3 t) + C$   
 22.  $x \sin x + C$  23.  $y^2 = (5x^2/2) + 3 \cos x - 3$  24.  $y^{3/2} = \frac{3}{2} \sin \pi x - \frac{1}{2}$  25. Yes; from the identities  $\cos^2 x + \sin^2 x = 1$ ,  
 $\cos 2x = \cos^2 x - \sin^2 x$ , we find that the constants have to satisfy the relations:  $C_2 = C_1 + 1, C_3 = C_1 + \frac{1}{2}$ .