[06-09-25-S12]

EXERCISES 2.5

1–4 ■ Find dy/dx and $dy/dx]_{x=1}$ in two ways: (a) using the Chain Rule and (b) without using the Chain Rule, as in Example 2.

1. $y = u^2, \quad u = x^2 + 2x + 3$

2. $y = u^2 - 2u + 3, \quad u = 5 - 6x$

3. $y = u^3, \quad u = x + (1/x)$

4. $y = u - u^2, \quad u = \sqrt{x} + \sqrt[3]{x}$

5–48 ■ Find the derivative of the function.

5. $F(x) = (x^2 + 4x + 6)^5$

6. $F(x) = (x^3 - 5x)^4$

7. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

8. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$

9. $f(t) = (2t^2 - 6t + 1)^{-8}$

10. $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4}$

11. $g(x) = \sqrt{x^2 - 7x}$

12. $k(x) = \sqrt[3]{1 + \sqrt{x}}$

13. $h(t) = \left(t - \dfrac{1}{t}\right)^{3/2}$

14. $F(s) = \sqrt{s^3 + 1}\,(s^2 + 1)^4$

15. $F(y) = \left(\dfrac{y - 6}{y + 7}\right)^3$

16. $s(t) = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$

17. $f(z) = \dfrac{1}{\sqrt[5]{2z - 1}}$

18. $f(x) = \dfrac{x}{\sqrt{7 - 3x}}$

19. $y = (2x - 5)^4(8x^2 - 5)^{-3}$

20. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

21. $y = \tan 3x$

22. $y = 4 \sec 5x$

23. $y = \cos(x^3)$

24. $y = \cos^3 x$

25. $y = (1 + \cos^2 x)^6$

26. $y = \tan(x^2) + \tan^2 x$

27. $y = \cos(\tan x)$

28. $y = \sin(\sin x)$

29. $y = \sec^2 2x - \tan^2 2x$

30. $y = \sqrt{1 + 2\tan x}$

31. $y = \csc(x/3)$

32. $y = \cot \sqrt[3]{1 + x^2}$

33. $y = \sin^3 x + \cos^3 x$

34. $y = \sin^2(\cos 4x)$

35. $y = \sin\dfrac{1}{x}$

36. $y = \dfrac{\sin^2 x}{\cos x}$

37. $y = \dfrac{1 + \sin 2x}{1 - \sin 2x}$

38. $y = x \sin\dfrac{1}{x}$

39. $y = \tan^2(x^3)$

40. $y = (\sin\sqrt{x^2 + 1}\,)^{\sqrt{2}}$

41. $y = \cos^2(\cos x) + \sin^2(\cos x)$

42. $y = \sin(\sin(\sin x))$

43. $y = \sqrt{x + \sqrt{x}}$

44. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

45. $f(x) = [x^3 + (2x - 1)^3]^3$

46. $g(t) = \sqrt[4]{(1 - 3t)^4 + t^4}$

47. $y = \sin(\tan\sqrt{\sin x}\,)$

48. $y = \sqrt{\cos(\sin^2 x)}$

49–52 ■ Find the equation of the tangent line to the curve at the given point.

49. $y = (x^3 - x^2 + x - 1)^{10}, \quad (1, 0)$

50. $y = \sqrt{x + (1/x)}, \quad (1, \sqrt{2})$

51. $y = \dfrac{8}{\sqrt{4 + 3x}}, \quad (4, 2)$

52. $y = \sin x + \cos 2x, \quad (\pi/6, 1)$

53. (a) Find an equation of the tangent line to the curve $y = \tan(\pi x^2/4)$ at the point $(1, 1)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

54. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a **bullet-nose curve.** Find an equation of the tangent line to this curve at the point $(1, 1)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

55. (a) If $f(x) = \sqrt{1 - x^2}/x$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

56. (a) If $f(x) = 1/(\cos^2 \pi x + 9\sin^2 \pi x)$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

57. Find all points on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

58. Find the x-coordinates of all points on the curve $y = \sin 2x - 2\sin x$ at which the tangent line is horizontal.

59. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$. Find $F'(3)$.

60. Suppose that $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Find $w'(0)$.

61. The displacement of a particle on a vibrating string is given by the equation

$$s(t) = 10 + \tfrac{1}{4}\sin(10\pi t)$$

where s is measured in centimeters and t in seconds. Find the velocity of the particle after t seconds.

62. If the equation of motion of a particle is given by $s = A\cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion.*
(a) Find the velocity of the particle at time t.
(b) When is the velocity 0?

63. A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by ± 0.35. In view of these data, the brightness of Delta Cephei at time t, where t is measured in days, has been modeled by the function

$$B(t) = 4.0 + 0.35\sin(2\pi t/5.4)$$

(a) Find the rate of change of the brightness after t days.
(b) Find, correct to two decimal places, the rate of increase after one day.

64. The frequency of vibrations of a vibrating violin string is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where L is the length of the string, T is its tension, and ρ is its linear density. [See Chapter 11 in D. E. Hall, *Musical Acoustics,* 2d ed. (Pacific Grove, CA: Brooks/Cole, 1991).] Find the rate of change of the frequency with respect to
(a) the length (when T and ρ are constant),
(b) the tension (when L and ρ are constant), and
(c) the linear density (when L and T are constant).

5. Let h be differentiable on $[0, \infty)$ and define G by $G(x) = h(\sqrt{x})$.
(a) Where is G differentiable?
(b) Find an expression for $G'(x)$.

6. Suppose f is differentiable on $\mathbb{R}$ and α is a real number. Let $F(x) = f(x^\alpha)$ and $G(x) = [f(x)]^\alpha$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$.

7. Suppose f is differentiable on $\mathbb{R}$. Let $F(x) = f(\cos x)$ and $G(x) = \cos(f(x))$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$.

8. If $g(t) = [f(\sin t)]^2$, where f is a differentiable function, find $g'(t)$.

9. If $g(x) = f(b + mx) + f(b - mx)$, where f is differentiable at b, find $g'(0)$.

0. Suppose $y = f(x)$ is a curve that always lies above the x-axis and never has a horizontal tangent, where f is differentiable everywhere. For what value of y is the rate of change of y^5 with respect to x eighty times the rate of change of y with respect to x?