[04-05-02-S12A]

5. $\left\{\sin \frac{n\pi}{2}\right\}$ **6.** $a_1 = 1,\ a_{n+1} = \frac{1}{1 + a_n}$

7–12 ■ Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

7. $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$ **8.** $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\}$

9. $\{1, 4, 7, 10, \ldots\}$ **10.** $\{\frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots\}$

11. $\{\frac{3}{2}, -\frac{9}{4}, \frac{27}{8}, -\frac{81}{16}, \ldots\}$ **12.** $\{0, 2, 0, 2, 0, 2, \ldots\}$

13–40 ■ Determine whether the sequence converges or diverges. If it converges, find the limit.

13. $a_n = \frac{1}{4n^2}$ **14.** $a_n = 4\sqrt{n}$

15. $a_n = \frac{n^2 - 1}{n^2 + 1}$ **16.** $a_n = \frac{4n - 3}{3n + 4}$

17. $a_n = \frac{n^2}{n + 1}$ **18.** $a_n = \frac{\sqrt[3]{n} + \sqrt[4]{n}}{\sqrt{n} + \sqrt[5]{n}}$

19. $a_n = (-1)^n \frac{n^2}{1 + n^3}$ **20.** $a_n = \frac{1}{5^n}$

21. $a_n = \cos(n\pi/2)$ **22.** $a_n = \sin(n\pi/2)$

23. $\left\{\frac{\pi^n}{3^n}\right\}$ **24.** $\{\arctan 2n\}$

25. $\left\{\frac{3 + (-1)^n}{n^2}\right\}$ **26.** $\left\{\frac{n!}{(n + 2)!}\right\}$

27. $\left\{\frac{\ln(n^2)}{n}\right\}$ **28.** $\{(-1)^n \sin(1/n)\}$

29. $\{\sqrt{n + 2} - \sqrt{n}\}$ **30.** $\left\{\frac{\ln(2 + e^n)}{3n}\right\}$

31. $a_n = n2^{-n}$ **32.** $a_n = \ln(n + 1) - \ln n$

33. $a_n = n^{-1/n}$ **34.** $a_n = (1 + 3n)^{1/n}$

35. $a_n = \frac{\cos^2 n}{2^n}$ **36.** $a_n = \frac{n \cos n}{n^2 + 1}$

37. $a_n = \frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{n}{n^2}$

38. $a_n = (\sqrt{n + 1} - \sqrt{n})\sqrt{n + \frac{1}{2}}$

39. $a_n = \frac{n!}{2^n}$ **40.** $a_n = \frac{(-3)^n}{n!}$

41–48 ■ Use a graph of the sequence to decide whether the sequence is convergent or divergent. If the sequence is convergent, guess the value of the limit from the graph and then prove your guess. (See the margin note on page 583 for advice on graphing sequences.)

41. $a_n = (-1)^n \frac{n + 1}{n}$ **42.** $a_n = 2 + (-2/\pi)^n$

43. $\left\{\arctan\left(\frac{2n}{2n + 1}\right)\right\}$ **44.** $\left\{\frac{\sin n}{\sqrt{n}}\right\}$

45. $a_n = \frac{n^3}{n!}$ **46.** $a_n = \sqrt[n]{3^n + 5^n}$

47. $a_n = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 1)}{(2n)^n}$

48. $a_n = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 1)}{n!}$

49. For what values of r is the sequence $\{nr^n\}$ convergent?

50. (a) If $\{a_n\}$ is convergent, show that

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$$

(b) A sequence $\{a_n\}$ is defined by $a_1 = 1$ and $a_{n+1} = 1/(1 + a_n)$ for $n \geq 1$. Assuming that $\{a_n\}$ is convergent, find its limit.

51–58 ■ Determine whether the given sequence is increasing, decreasing, or not monotonic.

51. $a_n = \frac{1}{3n + 5}$ **52.** $a_n = \frac{1}{5^n}$

53. $a_n = \frac{n - 2}{n + 2}$ **54.** $a_n = \frac{3n + 4}{2n + 5}$

55. $a_n = \cos(n\pi/2)$ **56.** $a_n = 3 + (-1)^n/n$

57. $a_n = \frac{n}{n^2 + n - 1}$ **58.** $a_n = \frac{\sqrt{n + 1}}{5n + 3}$

59. Find the limit of the sequence

$$\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\}$$

60. A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$.

(a) By induction, or otherwise, show that $\{a_n\}$ is increasing and bounded above by 3. Apply Theorem 10 to show that $\lim_{n\to\infty} a_n$ exists.

(b) Find $\lim_{n\to\infty} a_n$.

61. Show that the sequence defined by $a_1 = 1$, $a_{n+1} = 3 - 1/a_n$ is increasing and $a_n < 3$ for all n. Deduce that $\{a_n\}$ is convergent and find its limit.

62. Show that the sequence defined by $a_1 = 2$, $a_{n+1} = 1/(3 - a_n)$ satisfies $0 < a_n \leq 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.

63. (a) Fibonacci posed the following problem: Suppose that rabbits live forever and that every month each pair produces a new pair which becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the nth month? Show that the answer is f_n, where $\{f_n\}$ is the Fibonacci sequence defined in Example 2(c).

(b) Let $a_n = f_{n+1}/f_n$ and show that $a_{n-1} = 1 + 1/a_{n-2}$. Assuming that $\{a_n\}$ is convergent, find its limit.

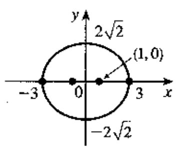
27. 18 **29.** $(2, \pm\pi/3)$ **31.** $(\pi - 1)/2$ **33.** $2(5\sqrt{5} - 1)$

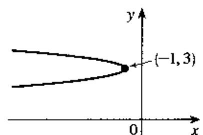
35. $\dfrac{2\sqrt{\pi^2+1} - \sqrt{4\pi^2+1}}{2\pi} + \ln\left[\dfrac{2\pi + \sqrt{4\pi^2+1}}{\pi + \sqrt{\pi^2+1}}\right]$

37. $471{,}295\pi/1024$

39. All curves have the vertical asymptote $x = 1$. For $c < -1$, the curve bulges to the right. At $c = -1$, the curve is the line $x = 1$. For $-1 < c < 0$, it bulges to the left. At $c = 0$ there is a cusp at $(0, 0)$. For $c > 0$, there is a loop.

41. $(\pm 1, 0)$, $(\pm 3, 0)$ **43.** $(-\frac{25}{24}, 3)$, $(-1, 3)$





45. $x^2 = 8(y - 4)$ **47.** $5x^2 - 20y^2 = 36$

49. $(x^2/25) + ((8y - 399)^2/160{,}801) = 1$

51. $r = 4/(3 + \cos\theta)$

53. $x = a(\cot\theta + \sin\theta\cos\theta)$, $y = a(1 + \sin^2\theta)$

APPLICATIONS PLUS ■ page 573

1. (b) $f(x) = (x^2 - L^2)/(4L) - (L/2)\ln(x/L)$ (c) No

3. (a) $2\pi r(r \pm d)$ (b) $\approx 3{,}360{,}000$ mi^2
(d) $\approx 78{,}400{,}000$ mi^2

5. (a) $y(t) = y_0/(1 - \varepsilon y_0^\varepsilon kt)^{1/\varepsilon}$ (b) $T = 1/(\varepsilon y_0^\varepsilon k)$
(c) ≈ 12.15 years

7. (a) $x = C(\theta - \frac{1}{2}\sin 2\theta)$ (b) Cycloid

9. (a) $P(z) = P_0 + g\int_0^z \rho(u)\,du$
(b) $(P_0 - \rho_0 gH)(\pi r^2) + \rho_0 gHe^{L/H}\int_{-r}^{r} e^{x/H} \cdot 2\sqrt{r^2 - x^2}\,dx$

11. (a) $y = (1/k)\cosh kx + a - 1/k$ or
$y = (1/k)\cosh kx - (1/k)\cosh kb + h$ (b) $(2/k)\sinh kb$

CHAPTER 10

Exercises 10.1 ■ page 586

1. $\{\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \ldots\}$ **3.** $\{1, \frac{3}{2}, \frac{5}{2}, \frac{35}{8}, \frac{63}{8}, \ldots\}$

5. $\{1, 0, -1, 0, 1, \ldots\}$ **7.** $a_n = 1/2^n$ **9.** $a_n = 3n - 2$

11. $a_n = (-1)^{n+1}(3/2)^n$ **13.** 0 **15.** 1

17. Diverges (to ∞) **19.** 0 **21.** Diverges

23. Diverges (to ∞) **25.** 0 **27.** 0 **29.** 0 **31.** 0

33. 1 **35.** 0 **37.** $\frac{1}{2}$ **39.** Diverges (to ∞)

41. Diverges **43.** $\pi/4$ **45.** 0 **47.** 0

49. $-1 < r < 1$ **51.** Decreasing **53.** Increasing

55. Not monotonic **57.** Decreasing **59.** 2

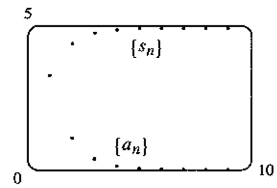
61. $(3 + \sqrt{5})/2$ **63.** (b) $(1 + \sqrt{5})/2$

65. (a) 0 (b) 9, 11

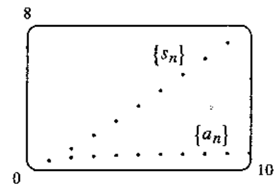
71. (b) $b_n = (1 + 2\cos\theta)/[1 + 2\cos(\theta/2^n)]$

Exercises 10.2 ■ page 596

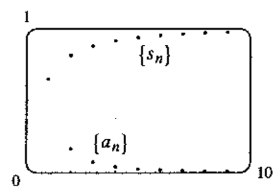
1. 3.33333, 4.44444, 4.81481, 4.93827, 4.97942, 4.99314, 4.99771, 4.99924, 4.99975, 4.99992
Convergent, sum = 5



3. 0.50000, 1.16667, 1.91667, 2.71667, 3.55000, 4.40714, 5.28214, 6.17103, 7.07103, 7.98012
Divergent (terms do not approach 0)



5. 0.64645, 0.80755, 0.87500, 0.91056, 0.93196, 0.94601, 0.95581, 0.96296, 0.96838, 0.97259
Convergent, sum = 1



7. $\frac{20}{3}$ **9.** $\frac{1}{2}$ **11.** 8 **13.** $5e/(3 - e)$ **15.** $\frac{8}{3}$

17. Divergent **19.** Divergent **21.** $\frac{1}{3}$ **23.** $\frac{17}{36}$

25. Divergent **27.** $\frac{3}{4}$ **29.** $\frac{3}{2}$ **31.** $\sin 1$ **33.** Divergent

35. Divergent **37.** $\frac{5}{9}$ **39.** $\frac{307}{999}$ **41.** 41,111/333,000

43. $2 < x < 4$, $1/(4 - x)$ **45.** $-5 < x < 5$, $x^2/[5(5 - x)]$

47. $|x - n\pi| < \pi/6$, n any integer, $1/(1 - 2\sin x)$ **49.** $\frac{1}{4}$

51. $a_1 = 0$, $a_n = 2/[n(n + 1)]$ for $n > 1$, sum = 1

53. (a) $S_n = D(1 - c^n)/(1 - c)$ (b) 5 **55.** $(\sqrt{3} - 1)/2$

57. $1/[n(n + 1)]$ **59.** The series is divergent

65. $\{s_n\}$ is bounded and increasing

67. (a) $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$

69. (a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}$; $[(n + 1)! - 1]/(n + 1)!$ (c) 1

Exercises 10.3 ■ page 603

Abbreviations: C, convergent; D, divergent

1. D **3.** C **5.** C **7.** D **9.** C **11.** D **13.** D

15. C **17.** C **19.** $p > 1$ **21.** $p < -1$ **23.** $(1, \infty)$

25. (a) 1.54977, error $\leqslant 0.1$ (b) 1.64522, error $\leqslant 0.005$
(c) $n > 1000$

27. 2.6124 **31.** $b < 1/e$

Exercises 10.4 ■ page 608

1. C **3.** C **5.** D **7.** C **9.** D **11.** C **13.** C

15. C **17.** C **19.** D **21.** C **23.** D **25.** C

27. C **29.** C **31.** D **33.** 0.567975, error $\leqslant 0.000\overline{3}$

35. 0.76352, error < 0.001 **45.** Yes