[06-11-11-T11]

Snell's Law

We tried to derive Snell's Law from Fermat's Principle that light follows the path of least time. We failed.

■ What went wrong

We expressed time as a function of x

$$t(x) = \frac{x}{c_1 \sin\theta_1} + \frac{d-x}{c_2 \sin\theta_2} \tag{1}$$

Wishing to find the path that minimized time, we found $\frac{\text{dt}}{\text{dx}}$ and set it equal to zero. This lead to $\frac{\sin\theta_2}{c_1} = \frac{\sin\theta_1}{c_2}$ which was wrong.

Remember when we began to look at the extremes ($x = 0,\ x = d$) in order to see if EQ1 made sense? So sure were we of EQ1, that we blew off looking at the extremes. We should have persisted. The diagrams on the following page make it pretty clear that the angles θ_1 and θ_2 are functions of x. So, the derivative of EQ1 was wrong.

■ Getting it right

$$T(x) = \frac{\sqrt{x^2+a^2}}{c_1} + \frac{\sqrt{b^2+(d-x)^2}}{c_2}$$

$$\frac{\text{dT}}{\text{dx}}(x) = \frac{x}{c_1\sqrt{x^2+a^2}} - \frac{d-x}{c_2\sqrt{b^2+(d-x)^2}}$$

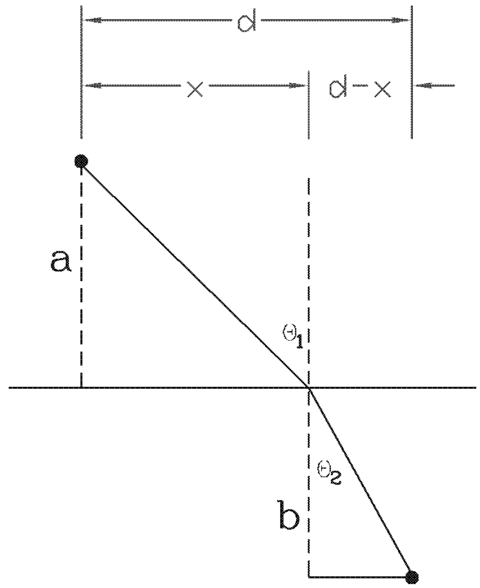
Note that $\frac{x}{\sqrt{x^2+a^2}} = \sin\theta_1$ and $\frac{d-x}{\sqrt{b^2+(d-x)^2}} = \sin\theta_2$. Then,

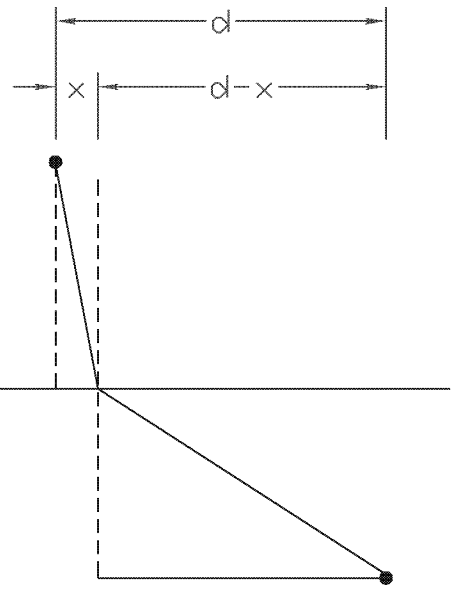
$$\frac{\text{dT}}{\text{dx}}(x) = \frac{\sin\theta_1}{c_1} - \frac{\sin\,\theta_2}{c_2}$$

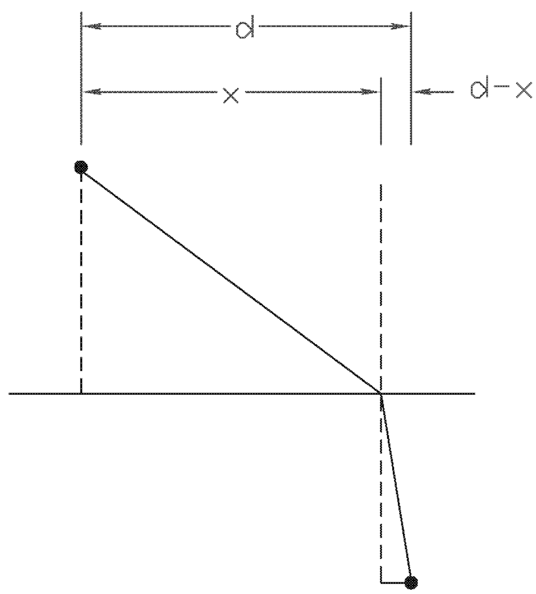
When $\frac{\text{dT}}{\text{dx}} = 0$,

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\,\theta_2}{c_2}$$

Which is Snell's Law


d
x
d−x
a
Θ_1
Θ_2
b


d
x
d−x


d
x
d−x