EXERCISE 3.3

1. Explain what is meant by the graph of a simple harmonic having a phase shift of -2.
2. Explain what is meant by the graph of a simple harmonic having a phase shift of 3.

In Problems 3–6, indicate the phase shift for each equation, and graph it over the stated region.

3. $y = \cos\left(x + \frac{\pi}{2}\right), \quad \frac{-\pi}{2} \le x \le \frac{3\pi}{2}$
4. $y = \cos\left(x - \frac{\pi}{2}\right), \quad \frac{\pi}{2} \le x \le \frac{5\pi}{2}$
5. $y = \sin\left(x - \frac{\pi}{4}\right), \quad -\pi \le x \le 2\pi$
6. $y = \cos\left(x + \frac{\pi}{4}\right), \quad -\pi \le x \le 2\pi$

In Problems 7–10, state the amplitude, period, and phase shift for each equation, and graph it over the indicated interval.

7. $y = 4\cos\left(\pi x + \frac{\pi}{4}\right), \quad -1 \le x \le 3$
8. $y = 2\sin\left(\pi x - \frac{\pi}{2}\right), \quad -2 \le x \le 2$
9. $y = -2\cos(2x + \pi), \quad -\pi \le x \le 3\pi$
10. $y = -3\sin(4x - \pi), \quad -\pi \le x \le \pi$
11. Graph

$$y = \cos\left(x - \frac{\pi}{2}\right) \quad \text{and} \quad y = \sin x$$

in the same coordinate system. Conclusion?

12. Graph

$$y = \sin\left(x + \frac{\pi}{2}\right) \quad \text{and} \quad y = \cos x$$

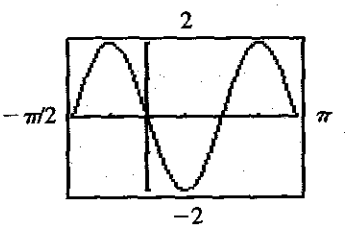
in the same coordinate system. Conclusion?

In Problems 13–16, graph each equation over the indicated interval.

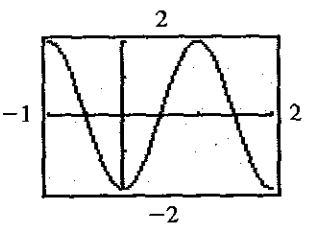
13. $y = -2 + 4\cos\left(\pi x + \frac{\pi}{4}\right), \quad -1 \le x \le 3$
14. $y = -3 + 2\sin\left(\pi x - \frac{\pi}{2}\right), \quad -2 \le x \le 2$
15. $y = 3 - 2\cos(2x + \pi), \quad -\pi \le x \le 3\pi$
16. $y = 4 - 3\sin(4x - \pi), \quad -\pi \le x \le \pi$

In Problems 17–20, match each equation with one of the fo lowing graphing utility displays. Explain how you made t choice relative to period and phase shift.

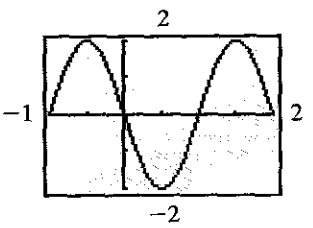
(A)



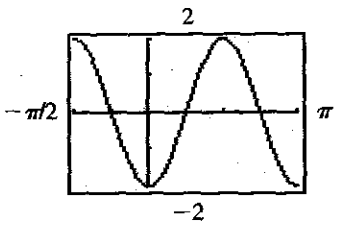
(B)



(C)

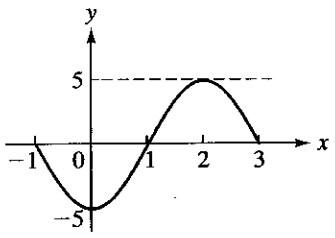


(D)



17. $y = 2\sin\left(\pi x - \frac{\pi}{2}\right)$
18. $y = 2\cos\left(\pi x + \frac{\pi}{2}\right)$
19. $y = 2\cos\left(2x + \frac{\pi}{2}\right)$
20. $y = 2\sin\left(2x - \frac{\pi}{2}\right)$

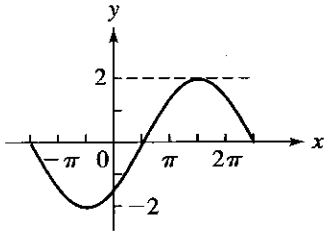
For Problems 21 and 22, refer to the graph:



21. If the graph is a graph of an equation of the form $y = A\sin(Bx + C)$, $0 < -C/B < 2$, find the equation.

22. If the graph is a graph of an equation of the form $y = A\sin(Bx + C)$, $-2 < -C/B < 0$, find the equation.

For Problems 23 and 24, refer to the graph:



23. If the graph is a graph of an equation of the form $y = A\cos(Bx + C)$, $-2\pi < -C/B < 0$, find the equation.

24. If the graph is a graph of an equation of the form $y = A\cos(Bx + C)$, $0 < -C/B < 2\pi$, find the equation.

C *In Problems 25 and 26, state the amplitude, period, and phase shift for each equation, and graph it over the indicated interval.*

25. $y = 2\sin\left(3x - \frac{\pi}{2}\right)$, $\frac{-2\pi}{3} \le x \le \frac{5\pi}{3}$

26. $y = -4\cos\left(4x + \frac{\pi}{2}\right)$, $\frac{-\pi}{2} \le x \le \pi$

In Problems 27 and 28, graph each equation over the indicated interval.

27. $y = 4 + 2\sin\left(3x - \frac{\pi}{2}\right)$, $\frac{-2\pi}{3} \le x \le \frac{5\pi}{3}$

28. $y = 6 - 4\cos\left(4x + \frac{\pi}{2}\right)$, $\frac{-\pi}{2} \le x \le \pi$

Problems 29–32 require the use of a graphing utility. First, state the amplitude, period, and phase shift of each function; then graph the function in a graphing utility.

29. $y = 2.3\sin\left[\frac{\pi}{1.5}(x - 2)\right]$, $0 \le x \le 6$

30. $y = -4.7\sin\left[\frac{\pi}{2.2}(x + 3)\right]$, $0 \le x \le 10$

31. $y = 18\cos[4\pi(x + 0.137)]$, $0 \le x \le 2$

32. $y = -48\cos[2\pi(x - 0.205)]$, $0 \le x \le 3$

Problems 33–40 require the use of a graphing utility. Graph the given equation and find the x intercept closest to the origin, correct to three decimal places. Use this intercept to find an equation of the form $y = A\sin(Bx + C)$ that has the same graph as the given equation.

33. $y = \sin x + \sqrt{3}\cos x$

34. $y = \sqrt{3}\sin x - \cos x$

35. $y = \sqrt{2}\sin x - \sqrt{2}\cos x$

36. $y = \sqrt{2}\sin x + \sqrt{2}\cos x$

37. $y = 1.4\sin 2x + 4.8\cos 2x$

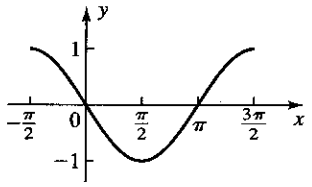
38. $y = 4.8\sin 2x - 1.4\cos 2x$

39. $y = 2\sin\frac{x}{2} - \sqrt{5}\cos\frac{x}{2}$

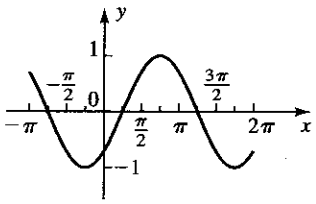
40. $y = \sqrt{5}\sin\frac{x}{2} + 2\cos\frac{x}{2}$

Exercise 3.3

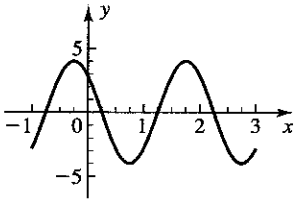
1. The graph with no phase shift is moved 2 units to the left.
3. Phase shift $= -\pi/2$



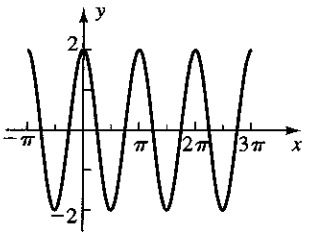
5. Phase shift $= \pi/4$



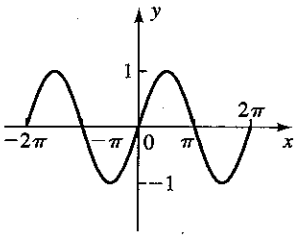
7. Amplitude $= 4$; Period $= 2$; Phase shift $= -\frac{1}{4}$



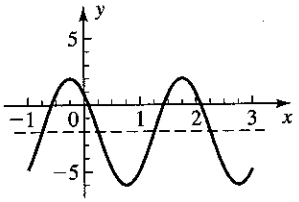
9. Amplitude $= 2$; Period $= \pi$; Phase shift $= -\pi/2$



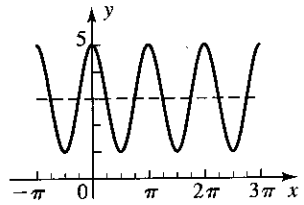
11. Both have the same graph; thus, $\cos(x - \pi/2) = \sin x$ for all x.



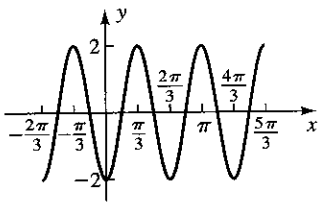
13.



15.



17. (B): The graph of the equation is a sine curve with a period of 2 and a phase shift of $\frac{1}{2}$, which means the sine curve is shifted $\frac{1}{2}$ unit to the right.
19. (A): The graph of the equation is a cosine curve with a period of π and a phase shift of $-\pi/4$, which means the cosine curve is shifted $-\pi/4$ unit to the left.
21. $y = 5\sin(\pi x/2 - \pi/2)$ **23.** $y = -2\cos(x/2 + \pi/4)$
25. Amplitude $= 2$; Period $= 2\pi/3$; Phase shift $= \pi/6$



27.