[07-03-29-MT-11]

Exercises 1.6

1. Recalling that $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$, develop expressions for
   a. $\tan(-x)$, b. $\sec(-x)$, c. $\cot(-x)$, d. $\csc(-x)$.
2. Use the information acquired in Exercise 1 to evaluate
   a. $\tan\left(-\frac{\pi}{4}\right)$, b. $\sec\left(-\frac{\pi}{6}\right)$, c. $\csc\left(-\frac{\pi}{3}\right)$,

   d. $\cot\left(-\frac{\pi}{4}\right)$, e. $\sec\left(-\frac{\pi}{3}\right)$, f. $\tan\left(-\frac{\pi}{6}\right)$,
   g. $\cot\left(-\frac{\pi}{3}\right)$, h. $\csc\left(-\frac{\pi}{4}\right)$, i. $\cot\left(-\frac{\pi}{6}\right)$,
   j. $\csc\left(-\frac{\pi}{6}\right)$, k. $\tan\left(-\frac{\pi}{3}\right)$, l. $\sec\left(-\frac{\pi}{4}\right)$.
3. Using the fact that $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$, develop expressions for
   a. $\tan(\pi - x)$, b. $\cot(\pi - x)$,
   c. $\sec(\pi - x)$, d. $\csc(\pi - x)$.
4. Use the information acquired in Exercise 3 to evaluate
   a. $\tan\frac{3\pi}{4}$, b. $\sec\frac{5\pi}{6}$, c. $\csc\frac{2\pi}{3}$,
   d. $\cot\frac{3\pi}{4}$, e. $\sec\frac{2\pi}{3}$, f. $\tan\frac{5\pi}{6}$,
   g. $\cot\frac{2\pi}{3}$, h. $\csc\frac{3\pi}{4}$, i. $\cot\frac{5\pi}{6}$,
   j. $\tan\frac{2\pi}{3}$, k. $\csc\frac{5\pi}{6}$, l. $\sec\frac{3\pi}{4}$.
5. Use the fact that $\cos(x + \pi) = -\cos x$ and $\sin(x + \pi) = -\sin x$, to develop expressions for
   a. $\tan(x + \pi)$, b. $\cot(x + \pi)$,
   c. $\sec(x + \pi)$, d. $\csc(x + \pi)$.
6. Use the information acquired in Exercise 5 to evaluate
   a. $\sec\frac{5\pi}{4}$, b. $\csc\frac{7\pi}{6}$, c. $\tan\frac{4\pi}{3}$,
   d. $\cot\frac{7\pi}{6}$, e. $\csc\frac{5\pi}{4}$, f. $\cot\frac{4\pi}{3}$,
   g. $\tan\frac{7\pi}{6}$, h. $\sec\frac{4\pi}{3}$, i. $\cot\frac{5\pi}{4}$,
   j. $\csc\frac{4\pi}{3}$, k. $\sec\frac{7\pi}{6}$, l. $\tan\frac{5\pi}{4}$.

In each of the following exercises, use the given information to evaluate all the remaining trigonometric functions.

Example

$\cos x = \frac{15}{17}$, $\sin x < 0$.

Solution

Since $\cos x > 0$ and $\sin x < 0$, x is in the fourth quadrant. $\sec x > 0$; the others are negative.

$$\sec x = \frac{1}{\cos x} = \frac{17}{15},$$

$$\sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \frac{225}{289}} = -\sqrt{\frac{64}{289}} = -\frac{8}{17},$$

$$\tan x = \frac{\sin x}{\cos x} = -\frac{8}{15},$$

$$\cot x = \frac{1}{\tan x} = -\frac{15}{8},$$

$$\csc x = \frac{1}{\sin x} = -\frac{17}{8}.$$

Example

$\tan x = -\frac{3}{4}$, $\sin x > 0$.

Solution

Since $\tan x < 0$, $\sin x > 0$, x is in the second quadrant. $\csc x > 0$; the others are negative.

$$\sec^2 x = \tan^2 x + 1 = \frac{9}{16} + 1 = \frac{25}{16},$$

$$\sec x = -\frac{5}{4}, \quad \text{since } \sec x < 0,$$

$$\cos x = -\frac{4}{5},$$

$$\cot x = -\frac{4}{3}.$$

Since $\frac{\sin x}{\cos x} = \tan x$,

$$\sin x = \cos x \tan x = \left(-\frac{4}{5}\right)\left(-\frac{3}{4}\right) = \frac{3}{5},$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{3}.$$

7. $\sec x = -\frac{13}{5}$, $\tan x > 0$.
8. $\csc x = \frac{2\sqrt{3}}{3}$, $\sec x > 0$.
9. $\tan x = \frac{1}{2}$, $\cos x > 0$.
10. $\sin x = \frac{5}{13}$, $\cot x < 0$.
11. $\cos x = \frac{2}{\sqrt{13}}$, $\tan x < 0$.
12. $\csc x = -\frac{17}{15}$, $\cos x < 0$.
13. $\cot x = \frac{1}{3}$, $\csc x < 0$.
14. $\sec x = \frac{5}{3}$, $\sin x > 0$.

Exercises 1.6

1. a. $\tan(-x) = -\tan x$.
   b. $\sec(-x) = \sec x$.
   c. $\cot(-x) = -\cot x$.
   d. $\csc(-x) = -\csc x$.
2. a. -1
   c. $\frac{-2}{\sqrt{3}}$ or $\frac{-2\sqrt{3}}{3}$
   e. 2
   g. $-\frac{1}{\sqrt{3}}$
   i. $-\sqrt{3}$
   k. $-\sqrt{3}$
3. a. $\tan(\pi - x) = -\tan x$.
   b. $\cot(\pi - x) = -\cot x$.
   c. $\sec(\pi - x) = -\sec x$.
   d. $\csc(\pi - x) = \csc x$.
4. a. -1
   c. $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
   e. -2
   g. $-\frac{1}{\sqrt{3}}$
   i. $-\sqrt{3}$
   k. 2
5. a. $\tan(x + \pi) = \tan x$.
   b. $\cot(x + \pi) = \cot x$.
   c. $\sec(x + \pi) = -\sec x$.
   d. $\csc(x + \pi) = -\csc x$.
6. a. $-\sqrt{2}$
   c. $\sqrt{3}$
   e. $-\sqrt{2}$
   g. $\frac{1}{\sqrt{3}}$
   i. 1
   k. $-\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$
7. $\sin x = -\frac{12}{13}$, $\cos x = -\frac{5}{13}$, $\tan x = \frac{12}{5}$, $\cot x = \frac{5}{12}$, $\csc x = -\frac{13}{12}$.
9. $\sin x = \frac{1}{\sqrt{5}}$, $\cos x = \frac{2}{\sqrt{5}}$, $\cot x = 2$, $\sec x = \frac{\sqrt{5}}{2}$, $\csc x = \sqrt{5}$.
11. $\sin x = -\frac{3}{\sqrt{13}}$, $\tan x = -\frac{3}{2}$, $\cot x = -\frac{2}{3}$, $\sec x = \frac{\sqrt{13}}{2}$, $\csc x = -\frac{\sqrt{13}}{3}$
13. $\sin x = -\frac{3}{\sqrt{10}}$, $\cos x = -\frac{1}{\sqrt{10}}$, $\tan x = 3$, $\sec x = -\sqrt{10}$, $\csc x = -\frac{\sqrt{10}}{3}$.

Review Exercises

1. a. $\frac{\sqrt{2}}{2}$ c. $\frac{1}{\sqrt{3}}$ e. 1 g. $\frac{1}{2}$ i. $\frac{1}{\sqrt{3}}$ k. $-\frac{2}{\sqrt{3}}$
12. a. $-\frac{\sqrt{2}}{2}$ c. 1 e. 1 g. $-\frac{\sqrt{3}}{2}$ i. 2