

[07-01-02-NAM-11]

Solution:

(a) $9^{\frac{1}{3}} \times 9^{\frac{1}{6}} = 9^{\frac{1}{3} + \frac{1}{6}}$	(b) $\frac{4^{\frac{3}{4}}}{4^{\frac{9}{4}}} = 4^{\frac{3}{4} - \frac{9}{4}}$
$= 9^{\frac{1}{2}}$	$= 4^{-\frac{3}{2}}$
$= \sqrt{9}$	$= (2^2)^{-\frac{3}{2}}$
$= \underline{\underline{3}}$	$= 2^{-3} = \underline{\underline{\frac{1}{8}}}$

(c) $8^{\frac{1}{2}} \times 2^{\frac{1}{2}} = (8 \times 2)^{\frac{1}{2}}$	(d) $12^3 \div 6^3 = \left(\frac{12}{6}\right)^3$
$= 16^{\frac{1}{2}}$	$= 2^3$
$= \sqrt{16}$	$= \underline{\underline{8}}$
$= \underline{\underline{4}}$	

Example 2 If $3^x = y$, express 3^{4-x} and 9^{x+1} in terms of y .

Solution:

$3^{4-x} = 3^4 \times 3^{-x}$	$9^{x+1} = 9^x \times 9^1$
$= 81 \left(\frac{1}{3^x}\right)$	$= 9(3^2)^x$
$= \frac{81}{y}$	$= 9(3^x)^2$
	$= \underline{\underline{9y^2}}$

[1.1]

1. Without using a calculator, evaluate:

(a) $(2^3 \times 3^2)^0$	(b) $8^{\frac{1}{2}} \times 8^{\frac{1}{6}}$	(c) $(2\sqrt{3})^4$
(d) $\frac{9^{\frac{1}{3}} \times 3^{\frac{1}{3}}}{6}$	(e) $25^{\frac{1}{4}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{6}}$	(f) $\frac{3^{\frac{1}{3}} \times 3^0 \times 9^{\frac{1}{3}}}{27^{\frac{2}{3}}}$

2. Given $y = 2x^{\frac{3}{2}}$, find

(a) y when $x = 9$,	(b) x when $y = 16$.
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3. If $2^x = y$, express the following in terms of y .

(a) 2^{x+3}	(b) 2^{2x-1}	(c) $3(2^{2-x})$
(d) $4(2^3)^{1-x}$	(e) $8(4^{x-2})$	(f) $8^x - 4^{-x}$

4. If $3^x = y$ and $2^x = z$, express the following in terms of y and/or z .

(a) $2(9^{x+1})$	(b) 6^{x-1}	(c) $9^{-x} \times 12^x$
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5. Show that $(8x^2)^{8-r} \left(\frac{1}{2x}\right)^r = 2^{24-4r} (x^{16-3r})$.

[1.2]

1. Solve the following equations.

(a) $3^{2x} = 27$

(b) $4^x = 32$

(c) $(\sqrt{2})^{3x} = \frac{1}{8}$

(d) $\left(\frac{1}{9}\right)^{x+2} = 3$

(e) $4^x(5^{2x}) = 10$

(f) $5^x - 25 = 0$

(g) $7^{x^2-4} - 1 = 0$

(h) $8^x = 4^{x+1}$

(i) $9^x = (\sqrt{3})^{x+2}$

(j) $\frac{4^x}{2^{x-1}} = 8^{2-x}$

(k) $2^{x^2} = 4^{2(x-1)}$

(l) $7^{x^2} = 49^{6-2x}$

(m) $4^{x^2-6} - 16^{x+1} = 0$

(n) $3^{x^2} = \frac{9^{x+1}}{27^x}$

2. Given that $y = ax^n - 23$, and that $y = 4$ when $x = 3$ and $y = 220$ when $x = 9$, find the value of a and of n .

3. Solve the following simultaneous equations.

(a) $5^x(25^{2y}) = 1$ and $3^{5x}(9^y) = \frac{1}{9}$

(b) $2(4^x) = 32^y$ and $\frac{125^x}{25^y} = 625$

(c) $\frac{3^x}{9^y} = 27$ and $4^{2x}(2^{6y}) = \frac{1}{4}$

4. Show that $\sqrt[n]{2 \times 4^m} = 2^{\frac{2m+1}{n}}$. Hence find the value of m and of n which satisfy the equations $\sqrt[n]{2 \times 4^m} = 8$ and $\frac{27^m}{9^{n+1}} = 81$ simultaneously.

5. By using the substitution $y = 2^x$, find the value of x such that $3(2^{x-1}) = 2^x + 4$.

6. By using the substitution $y = 3^x$, find the values of x such that

(a) $3^x = 4 - 3(3^{-x})$,

(b) $3^{2x} - 3^{x+2} = 3^x - 9$.

7. By using appropriate substitutions or otherwise, solve the following equations.

(a) $5^{2x} - 6(5^x) + 5 = 0$

(b) $2^{2x} - 10(2^x) + 16 = 0$

(c) $2(16^x) - 5(4^x) + 2 = 0$

(d) $7^{x+1} - 2 = 2(7^x) + 3$

(e) $9^{x+1} + 1 = 10(3^x)$

(f) $6(3^{x-1}) = 3^4 - 3^x$

8. Show that the equation $2^{2x+1} = 3(2^x) + 2$ is satisfied by only one value of x .

9. Solve the following equations.

(a) $8\left(\frac{1}{2}\right)^x + 2 = 2^x$

(b) $4^x = 2^{\frac{8}{x}}$

(c) $3^{x+1}(9^{2-x}) = \frac{1}{3}$

(d) $4^{x+\frac{1}{2}} - 2^{x+3} = 8 + 7(2^x)$

(e) $\frac{5^{x+1} - 20}{10 + 5^x} = 3$

(f) $2^x(5^x) = \frac{1}{100}(10^{x-1})^4$

10. If $\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}$ can be simplified to $\frac{k}{x^3}$, find the values of the constants r and k .

*11. Solve the simultaneous equations $64(4^y) = 16^x$ and $3^y = 4(3^{x-2}) - 1$.

*12. By using an appropriate substitution, find the value of x for which

$$x^{\frac{3}{2}} - 8x^{-\frac{3}{2}} = 7.$$

Exercise 1.1 (p. 2)

1. (a) 1 (b) 4 (c) 144 (d) $\frac{1}{2}$ (e) 5 (f) $\frac{1}{3}$
2. (a) 54 (b) 4
3. (a) $8y$ (b) $\frac{1}{2}y^2$ (c) $\frac{12}{y}$ (d) $\frac{32}{y^3}$ (e) $\frac{1}{2}y^2$ (f) $y^3 - \frac{1}{y^2}$
4. (a) $18y^2$ (b) $\frac{1}{6}yz$ (c) $\frac{z^2}{y}$

Exercise 1.2 (p. 5)

1. (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) -2 (d) $-\frac{5}{2}$ (e) $\frac{1}{2}$ (f) 2
(g) 2, -2 (h) 2 (i) $\frac{2}{3}$ (j) $\frac{5}{4}$ (k) 2 (l) $-6, 2$
(m) $-2, 4$ (n) $-2, 1$
2. $a = 3, n = 2$
3. (a) $x = -\frac{4}{9}, y = \frac{1}{9}$ (b) $x = 2, y = 1$ (c) $x = 1, y = -1$
4. $m = 4, n = 3$ 5. 3
6. (a) 0, 1 (b) 0, 2
7. (a) 0, 1 (b) 1, 3 (c) $-\frac{1}{2}, \frac{1}{2}$ (d) 0 (e) $-2, 0$ (f) 3
9. (a) 2 (b) ± 2 (c) 6 (d) 3 (e) 2 (f) 2
10. $r = 3, k = \frac{2}{3}$ 11. $x = 1, y = -1$ or $x = 2, y = 1$ 12. 4