

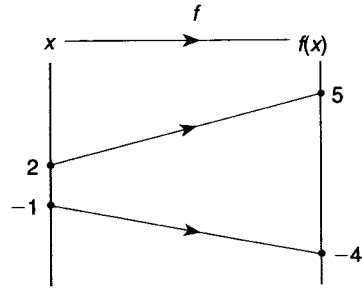
[07-01-01-NAM-11]

1. The figure shows part of the mapping

$$f: x \mapsto px + q.$$

Find

- (a) the value of p and of q ,
 (b) the image of 3 under f ,
 (c) the element whose image is 8.



2. Given that $f: x \mapsto 2 + \frac{a}{x}$, $x \neq 0$, and that $f^{-1}\left(\frac{1}{2}\right) = -2$, find the value of a . Find also the elements which are unchanged under f .

3. Given the functions $f: x \mapsto 2 - x$ and $g: x \mapsto \frac{3}{x}$ (for $x \neq 0$), find in similar form,
 (a) f^{-1} , (b) ff , (c) gg , (d) fg , (e) gfg .

4. Functions f and g are defined by

$$f: x \mapsto 4x - 3,$$

$$g: x \mapsto 2 - \frac{5}{x}, \quad x \neq 0.$$

Find an expression for the function

- (a) ff , (b) gf , (c) f^{-1} , (d) g^{-1} , (e) $(fg)^{-1}$.

5. Functions f and g are defined by

$$f: x \mapsto \frac{x-3}{2},$$

$$g: x \mapsto \frac{3-x}{x+1}, \quad x \neq -1.$$

Find

- (a) $f^{-1}(2)$, (b) $gf(-1)$, (c) $fgf(2)$.

Express in similar form, the function

- (d) g^{-1} , (e) fg , (f) gfg .

6. A function f is defined by $f: x \mapsto \frac{x}{1-x}$, $x \neq 1$.

- (a) Show that $f^2: x \mapsto \frac{x}{1-2x}$ for $x \neq \frac{1}{2}, 1$.

- (b) Obtain f^3 in a similar form.

7. A function h is defined by $h: x \mapsto \frac{x+3}{x-3}$ (for $x \neq 3$).

- (a) Show that $h(3+p) + h(3-p) = 2$ where p is a positive number.
 (b) Find the positive number q such that $h(q) = q - 1$.

8. Functions f and g are defined by

$$f: x \mapsto 4x - 3,$$

$$g: x \mapsto px + q, \text{ where } p > 0.$$

Find the values of p and q for which $gg(x) = f(x)$ for all values of x .

9. Functions f and g are defined by

$$f: x \mapsto ax + b, \text{ where } a \text{ and } b \text{ are constants,}$$

$$g: x \mapsto \frac{3}{x-2}, \quad x \neq 2.$$

Find expressions for fg and gf . Given that $fg(5) = 5$ and $g^{-1}(2) = f(2)$, calculate the value of a and of b .

10. The functions f and g are defined by

$$f: x \mapsto \frac{3x+2}{x}, \quad x \neq 0,$$

$$g: x \mapsto \frac{1}{x}, \quad x \neq 0.$$

Show that

(a) $f(x) = 2g(x) + 3,$

(b) $f^2(x) = 2gf(x) + 3.$

11. Sketch the graph and find the range of each of the following functions for the domain $-2 \leq x \leq 3$.

(a) $f: x \mapsto |2x + 1|$

(b) $g: x \mapsto x^2 - x - 2$

(c) $h: x \mapsto |x^2 - x|$

(d) $k: x \mapsto 3 - |x^2 - 2|$

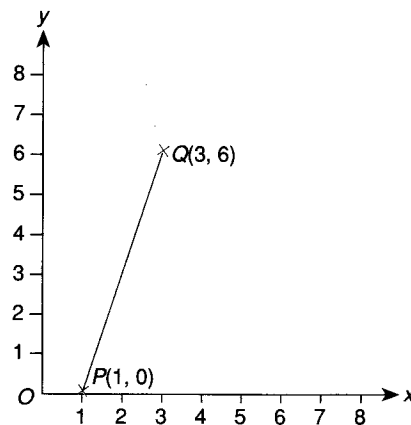
12. For each of the following functions, state the range that corresponds to the given domain.

(a) $f(x) = |2x - 9|$ for the domain $0 \leq x \leq 6$

(b) $g(x) = (x - 2)^2 - 3$ for the domain $0 \leq x \leq 5$

(c) $h(x) = |2 - 3(x - 1)^2|$ for the domain $-1 < x < 2$

13. The diagram shows the graph of $y = f(x)$. The scale is the same on each axis. Copy the diagram and on it sketch the graph of $y = f^{-1}(x)$, indicating the point on $y = f^{-1}(x)$ corresponding to the points P and Q . State the relationship between the graphs of $y = f(x)$ and $y = f^{-1}(x)$. Hence calculate the element whose images under f and f^{-1} are the same.



14. Functions f , g and h are defined by

$$\begin{aligned}f &: x \mapsto 1 - 2x, \\g &: x \mapsto |x + 1|, \\h &: x \mapsto x^2 - 2.\end{aligned}$$

- (a) Evaluate $gh(-4)$ and $f^{-1}gh(-4)$.
- (b) Find in similar form, gf , fg and gh .
- (c) Find the elements whose images under fg are -3 .
- (d) Show that $hg : x \mapsto x^2 + 2x - 1$.

15. Functions f and g are defined by $f : x \mapsto \frac{x+2}{x-2}$, $x \neq 2$, and $g : x \mapsto mx + c$, where m and c are constants.

- (a) Obtain an expression for f^{-1} .
- (b) Given that $g^{-1}(2) = f(3)$ and $gf^{-1}(2) = 5$, find the value of m and of c .

16. A function f is defined by $f : x \mapsto |x^2 - a| + b$ where a and b are constants. Given that $f(-2) = b$ and $f(3) = 8$, find the value of a and of b . Sketch the graph of f for $-2 \leq x \leq 3$ and state the corresponding range.

17. Functions f and g are defined by

$$\begin{aligned}f &: x \mapsto e^{x-2}, \\g &: x \mapsto \frac{1}{3} \ln x, \quad x > 0.\end{aligned}$$

- (a) Show that $f^{-1} : x \mapsto \ln x + 2$, (for $x > 0$).
- (b) Find in similar form, g^{-1} and gf .

Miscellaneous Exercise 8 (p. 151)

1. (a) $p = 3, q = -1$ (b) 8 (c) 3 2. $a = 3; x = -1$
3. (a) $f^{-1}: x \mapsto 2 - x$ (b) $ff: x \mapsto x$ (c) $gg: x \mapsto x, x \neq 0$
- (d) $fg: x \mapsto \frac{2x-3}{x}, x \neq 0$ (e) $gfg: x \mapsto \frac{3x}{2x-3}, x \neq 0, \frac{3}{2}$
4. (a) $ff: x \mapsto 16x - 15$ (b) $gf: x \mapsto \frac{8x-11}{4x-3}, x \neq \frac{3}{4}$
- (c) $f^{-1}: x \mapsto \frac{1}{4}(x+3)$ (d) $g^{-1}: x \mapsto \frac{5}{2-x}, x \neq 2$
- (e) $(fg)^{-1}: x \mapsto \frac{20}{5-x}, x \neq 5$
5. (a) 7 (b) -5 (c) 2 (d) $g^{-1}: x \mapsto \frac{3-x}{x+1}, x \neq -1$
- (e) $fg: x \mapsto -\frac{2x}{x+1}, x \neq -1$ (f) $gfg: x \mapsto \frac{5x+3}{1-x}, x \neq -1, 1$
6. $f^3: x \mapsto \frac{x}{1-3x}, x \neq 1, \frac{1}{2}, \frac{1}{3}$ 7. (b) $q = 5$ 8. $p = 2, q = -1$.
9. $fg: x \mapsto \frac{3a}{x-2} + b, x \neq 2; gf: x \mapsto \frac{3}{ax+b-2}, x \neq \frac{2-b}{a}; a = -\frac{3}{2}, b = \frac{13}{2}$
11. (a) $0 \leq f(x) \leq 7$ (b) $-2\frac{1}{4} \leq g(x) \leq 4$ (c) $0 \leq h(x) \leq 6$ (d) $-4 \leq k(x) \leq 3$
12. (a) $0 \leq f(x) \leq 9$ (b) $-3 \leq g(x) \leq 6$ (c) $0 \leq h(x) < 10$
13. The graphs are reflective images of each other in the line $y = x; 1\frac{1}{2}$
14. (a) 15, -7 (b) $gf: x \mapsto |2 - 2x|; fg: x \mapsto 1 - 2|x + 1|; gh: x \mapsto |x^2 - 1|$
- (c) -3, 1
15. (a) $f^{-1}: x \mapsto \frac{2(x+1)}{x-1}, x \neq 1$ (b) $m = 3, c = -13$
16. $a = 4, b = 3, 3 \leq y \leq 8$
17. (b) $g^{-1}: x \mapsto e^{3x}; gf: x \mapsto \frac{1}{3}(x-2)$