06-10-16-T11B

Division of a line segment parallel to x-axis or to y-axis

Dividing Points [J10 3.1.1] pps. 125-126

■ Internal and external dividing points

When P lies between A and B, we say "P divides AB internally" and "P is an internal dividing point of AB".

A P B

Figure 1

When P lies beyond A or B, we say "P divides AB externally" and "P is an external dividing point of AB".

A B P

Figure 2

P A B

Figure 3

■ The ratio in which P divides AB

Whether the division of AB by P is internal or external, we can talk about the ratio into which P divides AB. In each case, this is the ratio of the distances of P from the endpoints of AB. In the following discussion, points A, B, P have, respectively, coordinates a, b, p.

In each of the figures above,

the ratio into which P divides AB is $\frac{\text{distance AP}}{\text{distance BP}} = \frac{|\text{AP}|}{|\text{BP}|}$.

■ Internally divided

In figure 1: $a < p < b$. So, $\frac{|\text{AP}|}{|\text{BP}|} = \frac{p-a}{b-p}$.

■ Externally divided

In figure 2: $a < b < p$. So, $\frac{|AP|}{|BP|} = \frac{p-a}{p-b}$.

In figure 3: $p < a < b$. So, $\frac{|AP|}{|BP|} = \frac{a-p}{b-p} = \frac{p-a}{p-b}$.

■ Theorems

◆ Thm 1. Given two points $A(a)$ and $B(b)$. If $P(p)$ divides AB internally in the ratio $\frac{m}{n}$, then $p = \frac{m\,b+n\,a}{m+n}$.

Proof.

$$\frac{m}{n} = \frac{|AP|}{|BP|} = \frac{p-a}{b-p} \Longrightarrow m(b-p) = n(p-a) \Longrightarrow m\,b - m\,p = n\,p - n\,a \Longrightarrow m\,b + n\,a = n\,p + m\,p \Longrightarrow p = \frac{m\,b+n\,a}{m+n}.$$

□

◆ Thm 2. Given two points $A(a)$ and $B(b)$. If $P(p)$ divides AB externally in the ratio $\frac{m}{n}$, then $p = \frac{m\,b-n\,a}{m-n}$.

$$\frac{m}{n} = \frac{|AP|}{|BP|} = \frac{p-a}{p-b} \Longrightarrow m(p-b) = n(p-a) \Longrightarrow m\,p - m\,b = n\,p - n\,a \Longrightarrow m\,p - n\,p = m\,b - n\,a \Longrightarrow p = \frac{m\,b-n\,a}{m-n}.$$

□

Note that if P is the midpoint of AB then P divides AB internally into two equal segments. Thus, $\frac{m}{n} = 1 \Longrightarrow m = n$. By theorem 1,

$$p = \frac{m\,b+m\,a}{m+m} = \frac{b+a}{2}$$

■ Examples

[EX1] Given $A(2)$ and $B(9)$. Find point $P(p)$ that internally divides AB at a ratio of $\frac{3}{4}$.

$p = \frac{m\,b+n\,a}{m+n}$, where $\frac{m}{n} = \frac{3}{4}$. Thus, $p = \frac{3\,(9)+4\,(2)}{7} = \frac{35}{7} = 5$.

[EX2] Given $A(2)$ and $B(5)$. Find point $P(p)$ that externally divides AB at a ratio of $\frac{7}{4}$.

$p = \frac{m\,b-n\,a}{m-n}$, where $\frac{m}{n} = \frac{7}{4}$. Thus, $p = \frac{7\,(5)-4\,(2)}{7-4} = \frac{27}{3} = 9$.

[EX3] Given $A(5)$ and $B(9)$. Find point $P(p)$ that externally divides AB at a ratio of $\frac{3}{7}$.

$p = \frac{m\,b-n\,a}{m-n}$, where $\frac{m}{n} = \frac{3}{7}$. Thus, $p = \frac{3\,(9)-7\,(5)}{3-7} = \frac{-8}{-4} = 2$.

Dividing Points Practice Problems (with answers)

■ A. Given points $A(-3)$, $B(9)$, find the following

[1] The number p such that point $P(p)$ that divides AB internally in the ratio $3:4$. ANS: $\frac{15}{7}$

[2] The number p such that point $P(p)$ that divides AB externally in the ratio $3:4$. ANS: -39

[3] The number p such that point $P(p)$ that divides AB internally in the ratio $2:5$. ANS: $\frac{3}{7}$

[4] The number p such that point $P(p)$ that divides AB externally in the ratio $2:5$. ANS: -11

■ B. Given point $A(3)$ find the following

[1] The number b such that point $P(2)$ divides AB internally in the ratio $2:5$. ANS: $\frac{-1}{2}$

[2] The number b such that point $P(5)$ divides AB externally in the ratio $2:5$. ANS: 0

[3] The number b such that point $P(2)$ divides AB internally in the ratio $3:2$. ANS: $\frac{4}{3}$

[4] The number b such that point $P(-3)$ divides AB externally in the ratio $7:2$. ANS: $\frac{-9}{7}$