

[06-09-11-T11]
Quadratic Inequalities

■ For all questions on this problem set, answer using *both* interval notation and statements such as $-5 \leq x < 5$ or $x < -8 \vee x > 2$.

■ A. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 - x - 12 \leq 0$

[2] $x^2 - x - 12 > 0$

[3] $x^2 + 2x - 15 \geq 0$

[4] $x^2 + 2x - 15 < 0$

[5] $x^2 - 11x + 28 \leq 0$

[6] $x^2 - 11x + 28 \geq 0$

■ B. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 - 5 > 0$

[2] $x^2 - 5 < 0$

[3] $x^2 - 49 \leq 0$

[4] $x^2 - 49 \geq 0$

[5] $x^2 < 9$

[6] $-x^2 + 9 < 0$

■ C. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 + 14x + 49 < 0$

[2] $x^2 + 14x + 49 > 0$

[3] $x^2 > 8x - 15$

[4] $x^2 - 8x > -15$

[5] $2x^2 - 22x + 60 > 0$

[6] $2(x - 5)(x - 6) < 0$

■ D. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 - 4x + 3 > 0$

[2] $x^2 - 4x + 3 < 0$

[3] $x^2 - 22x + 121 \geq 0$

[4] $x^2 - 22x + 121 \leq 0$

[5] $x^2 + 2\sqrt{2}x + 2 < 0$

[6] $x^2 + 2\sqrt{2}x + 2 > 0$

■ E. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 + 1 < 0$

[2] $x^2 + 1 \leq 0$, careful. The instructions at the top ask for all $x \in \mathbb{R}$.

[3] $x^2 + x - 5 \geq 0$

[4] $x^2 + x - 5 < 0$

[5] $2x^2 + x - 2 < 0$

[6] $2x^2 + x - 2 \geq 0$

■ F. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 + x + 3 \geq 0$

[2] $4x^2 \geq -x - 2$

[3] $x^2 - 4x + 3 \geq 0$

[4] $x^2 - 4x + 4 < 0$

[5] $x^2 - 4x + 5 < 0$

[6] $x^2 - 3x + 5 \geq -x^2 + 2x$

■ G. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x^2 + x + 3 \geq 0$

[2] $0 \geq x^2$

[3]
$$\begin{cases} x^2 - 9x + 20 > 0 \\ x^2 - 6x + 9 < 0 \end{cases}$$

$$[4] \quad \begin{cases} x^2 < 5 \\ x^2 > 0 \end{cases}$$

$$[5] \quad x^2 < 5 \vee x^2 < 0$$

$$[6] \quad x^2 < 5 \wedge x^2 < 0$$

$$[7] \quad x^2 < 0 \vee x^2 > 0$$

$$[8] \quad x^2 \leq 0 \wedge x^2 \geq 0$$

Answers to [06-09-11-T11]

■ A. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $-3 \leq x \leq 4$ [2] $x < -3 \vee x > 4$ [3] $x \leq -5 \vee x \geq 3$
[4] $-5 < x < 3$ [5] $4 \leq x \leq 7$ [6] $x \leq 4 \vee x \geq 7$

■ B. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x < -\sqrt{5} \vee x > \sqrt{5}$ [2] $-\sqrt{5} < x < \sqrt{5}$
[3] $-7 \leq x \leq 7$ [4] $x \leq -7 \vee x \geq 7$
[5] $-3 < x < 3$ [6] $x < -3 \vee x > 3$

■ C. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $\{ \}$ [2] $x < -7 \vee x > -7$ [3] $x < 3 \vee x > 5$
[4] $x < 3 \vee x > 5$ [5] $x < 5 \vee x > 6$ [6] $5 < x < 6$

■ D. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $x < 1 \vee x > 3$ [2] $1 < x < 3$ [3] \mathbb{R}
[4] $x = 11$ [5] $\{ \}$ [6] $x < -\sqrt{2} \vee x > -\sqrt{2}$

■ E. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] $\{ \}$ [2] $\{ \}$ [3] $x \leq \frac{1}{2}(-1 - \sqrt{21}) \vee x \geq \frac{1}{2}(-1 + \sqrt{21})$
[4] $\frac{1}{2}(-1 - \sqrt{21}) < x < \frac{1}{2}(-1 + \sqrt{21})$ [5] $\frac{1}{4}(-1 - \sqrt{17}) < x < \frac{1}{4}(-1 + \sqrt{17})$
[6] $x \leq \frac{1}{4}(-1 - \sqrt{17}) \vee x \geq \frac{1}{4}(-1 + \sqrt{17})$

■ F. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] \mathbb{R} [2] \mathbb{R} [3] $x \leq 1 \vee x \geq 3$
[4] \emptyset [5] \emptyset [6] \mathbb{R}

■ G. Find all $x \in \mathbb{R}$ that satisfy the given conditions.

[1] \mathbb{R} [2] $x = 0$ [3] \emptyset
[4] $-\sqrt{5} < x < 0 \vee 0 < x < \sqrt{5}$
[5] $-\sqrt{5} < x < \sqrt{5}$ [6] \emptyset
[7] $x < 0 \vee x > 0$ [8] $x = 0$