06-10-16-T11A

Division of a line segment in the plane

■ Coordinates of internal and external dividing points

■ Formulae

Let AB be any line segment in the coordinate plane and P a point that divides AB in into segments AP and PB such that the ratio of AP to PB is $m : n$; that is, $\frac{\text{AP}}{\text{PB}} = \frac{m}{n}$. If $A(x_1, y_1)$ and $B(x_2, y_2)$, determine the coordinates of $P(x, y)$.

Figure 1 represents the situation described above.

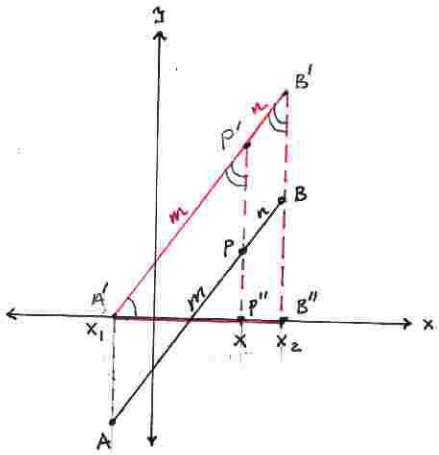

Figure 1

Imagine that we translate $\overline{\text{APB}}$ to position $A'P'B'$. Note that $\Delta A'P''P' \approx \Delta A'B''B'$. Then $\frac{A'P'}{A'P''} = \frac{A'B'}{A'B''}$. Or, equivalently

$$\frac{m}{x-x_1} = \frac{m+n}{x_2-x_1} \Longrightarrow x - x_1 = \frac{m(x_2-x_1)}{m+n} \Longrightarrow x = \frac{m(x_2-x_1)}{m+n} + x_1 \Longrightarrow x = \frac{m\,x_2 - m\,x_1 + m\,x_1 + n\,x_1}{m+n}$$

$$x = \frac{m\,x_2 + n\,x_1}{m+n}$$

A similar treatment can be given the y-compenent of P.

Therefore, if $A(x_1, y_1)$ and $B(x_2, y_2)$, and $P(x, y)$ divides AB in the ratio AP : PB = $m : n$ then

$$\mathbf{x = \frac{m\,x_2 + n\,x_1}{m+n},\ y = \frac{m\,y_2 + n\,y_1}{m+n},\ \text{where } P(x, y) \text{ internally divides AB}} \qquad \mathbf{(1)}$$

A similar derivation can be given for the coordinates of a point that externally divides AB; that is, where AP = AB + BP.

$$\mathbf{x = \frac{m\,x_2 - n\,x_1}{m-n},\ y = \frac{m\,y_2 - n\,y_1}{m-n},\ \text{where } P(x, y) \text{ externally divides AB}} \qquad \mathbf{(2)}$$

■ Generality

The formulae of (1) and (2) are general. When the line segment is parallel to an axis, for example the x-axis, y-coordinates of the points A, B, and P are zero. Thus, only the formula for the x-coordinate is used. When P is the midpoint of AB, the equations of (1) reduce to the familiar midpoint formulae: $x = \frac{x_2+x_1}{2}$, $y = \frac{y_2+y_1}{2}$.

In[150]:= $x[m\_, n\_, a\_, b\_] = \frac{m\,b + n\,a}{m + n}$