P. 55

[1] [1.1] $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

Proof: $\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$
$= \cos^2\alpha - \sin^2\alpha. \quad \alpha \in \mathbb{R}.$

[1.2] $\cos 2\alpha = 1 - 2\sin^2\alpha$

Proof: $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$= \cos^2\alpha - \sin^2\alpha + \sin^2\alpha - \sin^2\alpha$
$= 1 - 2\sin^2\alpha$

[1.3] $\cos 2\alpha = 2\cos^2\alpha - 1$

proof
$= \cos^2\alpha - \sin^2\alpha$
$= \cos^2\alpha - \sin^2\alpha + \cos^2\alpha - \cos^2\alpha$
$= 2\cos^2\alpha - 1$

[1.4] $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$, $\alpha \neq \frac{\pi}{2} + n\pi$, $\alpha \neq \frac{\pi}{4} + n\frac{\pi}{2}$, $n \in \mathbb{Z}$

Proof $\tan 2\alpha = \tan(\alpha+\alpha)$

$$= \frac{\tan\alpha + \tan\alpha}{1-\tan^2\alpha} = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

→

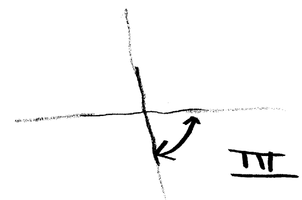
[2.1] $(\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\alpha \quad , \alpha \in \mathbb{R}$

$$\begin{aligned} LHS &= (\sin\alpha + \cos\alpha)^2 \\ &= \sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha \\ &= 1 + 2\sin\alpha\cos\alpha \\ \therefore \quad &= 1 + \sin 2\alpha \end{aligned}$$

[2.2] $\cos^4\alpha - \sin^4\alpha = \cos 2a$

$$\begin{aligned} LHS &= \cos^4\alpha - \sin^4\alpha \\ &= (\cos^2\alpha + \sin^2\alpha)(\cos^2\alpha - \sin^2\alpha) \\ \therefore \quad &= \cos 2\alpha \end{aligned}$$

P 56



[3] $\sin\alpha = -\frac{1}{2}$. $-\frac{\pi}{2} < \alpha < 0$.

[3.1] $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\begin{aligned} \cos\alpha &= \sqrt{1 - \left(\tfrac{1}{4}\right)} \\ &= \sqrt{\tfrac{3}{4}} \end{aligned}$$

$$\boxed{\cos\alpha = \frac{\sqrt{3}}{2}}$$

$$\sin 2\alpha = 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\sin 2\alpha = -\frac{\sqrt{3}}{2}$$

→

[3.2] $\cos 2\alpha = 1 - 2\sin^2\alpha$

$= 1 - 2\left(\frac{1}{4}\right)$

$= 1 - \frac{1}{2}$

$\boxed{\cos 2\alpha = \frac{1}{2}}$

[3.3] $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\tan\alpha = \dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\dfrac{1}{\sqrt{3}}$

$\tan 2\alpha = \dfrac{2\left(-1/\sqrt{3}\right)}{1 - \left(-\frac{1}{\sqrt{3}}\right)^2}$ $\quad 1 - \frac{1}{3}$

$= \dfrac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= -\dfrac{3}{\sqrt{3}} = -\dfrac{3\sqrt{3}}{3} = -\sqrt{3}$

$\boxed{\tan 2\alpha = -\sqrt{3}}$

$\longrightarrow$

p 56

[4.1] $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

<u>Proof</u>:

$$
\begin{aligned}
\sin 3\alpha &= \sin(2\alpha + \alpha) \\
&= \sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha \\
&= 2\sin\alpha\cos\alpha\cos\alpha + (1 - 2\sin^2\alpha)\sin\alpha \\
&= 2\sin\alpha\cos^2\alpha + \sin\alpha - 2\sin^3\alpha \\
&= 2\sin\alpha(1 - \sin^2\alpha) + \sin\alpha - 2\sin^2\alpha \\
&= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^2\alpha \\
\therefore \quad &= 3\sin\alpha - 4\sin^3\alpha
\end{aligned}
$$

$\square$

[4.2] $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$$
\begin{aligned}
\cos 3\alpha &= \cos(2\alpha + \alpha) \\
&= \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha \\
&= (2\cos^2\alpha - 1)\cos\alpha - 2\sin\alpha\cos\alpha\sin\alpha \\
&= 2\cos^3\alpha - \cos\alpha - 2\sin^2\alpha\cos\alpha \\
&= 2\cos^3\alpha - \cos\alpha - 2(1 - \cos^2\alpha)\cos\alpha \\
&= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha \\
\therefore \quad &= 4\cos^3\alpha - 3\cos\alpha
\end{aligned}
$$

$\square$