Chapter 1

Exercises 1.1

1. a. II c. IV e. I g. IV
2. a. $\frac{7\pi}{3}\ \frac{13\pi}{3}\ -\frac{5\pi}{3}\ -\frac{11\pi}{3}$ c. $\frac{11\pi}{4}\ \frac{19\pi}{4}\ -\frac{5\pi}{4}\ -\frac{13\pi}{4}$
   e. $\frac{7\pi}{6}\ \frac{19\pi}{6}\ -\frac{17\pi}{6}\ -\frac{29\pi}{6}$
3. a. $-\frac{\pi}{6}$ b. $\frac{5\pi}{6}$ c. $\frac{7\pi}{6}$
5. a. $-\frac{3\pi}{4}$ b. $\frac{\pi}{4}$ c. $\frac{7\pi}{4}$
7. a. $\frac{3\pi}{7}$ b. $\frac{4\pi}{7}$ c. $\frac{10\pi}{7}$
9. a. $\frac{\pi}{12}$ b. $-\frac{11\pi}{12}$ c. $\frac{11\pi}{12}$

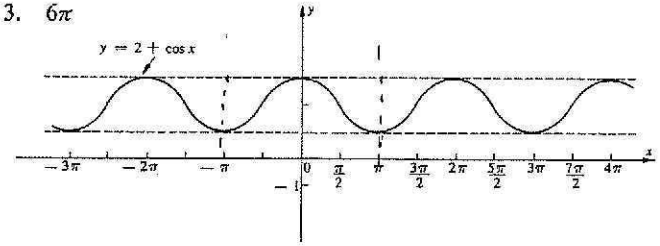
Exercises 1.2

1. a. $(u, -v)$ b. u c. $-v$
2. a. $\frac{1}{2}$ c. $\frac{\sqrt{3}}{2}$ e. $-\frac{\sqrt{2}}{2}$
3. a. $(-u, -v)$ b. $-u$ c. $-v$
4. a. $-\frac{\sqrt{3}}{2}$ c. $-\frac{1}{2}$ e. $-\frac{\sqrt{2}}{2}$
5. a. $(-u, v)$ b. $-u$ c. v
6. a. $-\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{3}}{2}$ e. $\frac{\sqrt{3}}{2}$

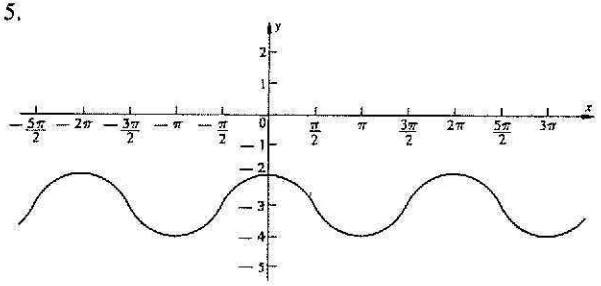
9. $\frac{8}{17}$ 11. $\frac{2\sqrt{2}}{3}$ 13. $\frac{15}{17}$ 15. $-\frac{\sqrt{21}}{5}$ 17. $\frac{\sqrt{5}}{3}$

Exercises 1.3

3. 6π

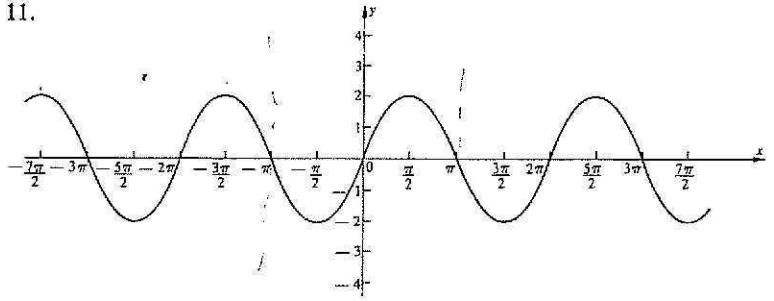


5.



9. a. $\frac{2\pi}{3}$ c. 4π e. 6π

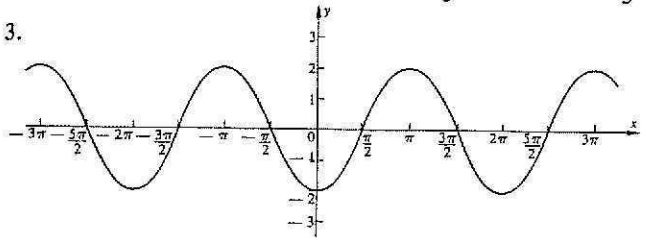
11.



Exercises 1.4

1. a. $\frac{1}{3}$ c. 2 e. $\frac{4}{3}$ g. $\frac{9}{5}$ i. 6

3.



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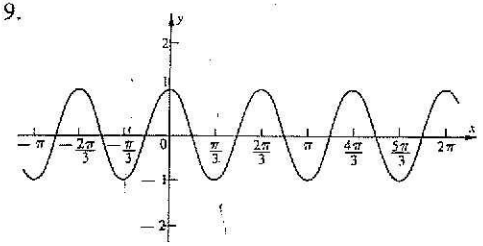
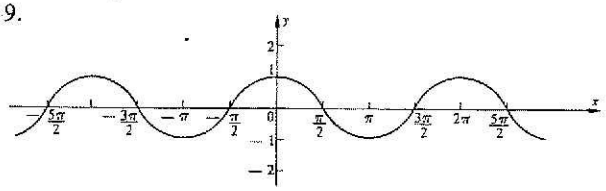


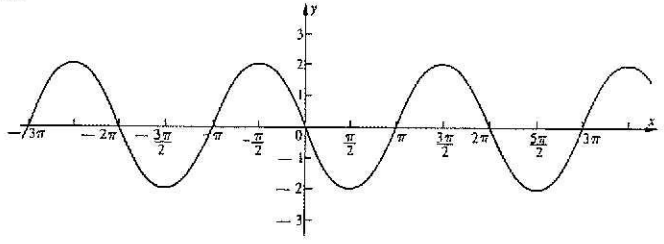
Exercises 1.5

1. $1, -\frac{\pi}{2}, 2\pi$
3. $1, -\frac{\pi}{2}, \pi$
5. $2, -\pi, 2\pi$
7. $3, \frac{\pi}{4}, \pi$

9.



13.



Exercises 1.6

1. a. $\tan(-x) = -\tan x$.
   b. $\sec(-x) = \sec x$.
   c. $\cot(-x) = -\cot x$.
   d. $\csc(-x) = -\csc x$.
2. a. -1
   c. $\frac{-2}{\sqrt{3}}$ or $\frac{-2\sqrt{3}}{3}$
   e. 2
   g. $-\frac{1}{\sqrt{3}}$
   i. $-\sqrt{3}$
   k. $-\sqrt{3}$
3. a. $\tan(\pi - x) = -\tan x$.
   b. $\cot(\pi - x) = -\cot x$.
   c. $\sec(\pi - x) = -\sec x$.
   d. $\csc(\pi - x) = \csc x$.
4. a. -1
   c. $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
   e. -2
   g. $-\frac{1}{\sqrt{3}}$

   i. $-\sqrt{3}$
   k. 2
5. a. $\tan(x + \pi) = \tan x$.
   b. $\cot(x + \pi) = \cot x$.
   c. $\sec(x + \pi) = -\sec x$.
   d. $\csc(x + \pi) = -\csc x$.
6. a. $-\sqrt{2}$
   c. $\sqrt{3}$
   e. $-\sqrt{2}$
   g. $\frac{1}{\sqrt{3}}$
   i. 1
   k. $-\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$
7. $\sin x = -\frac{12}{13}, \cos x = -\frac{5}{13}, \tan x = \frac{12}{5}, \cot x = \frac{5}{12}, \csc x = -\frac{13}{12}$.
9. $\sin x = \frac{1}{\sqrt{5}}, \cos x = \frac{2}{\sqrt{5}}, \cot x = 2, \sec x = \frac{\sqrt{5}}{2}, \csc x = \sqrt{5}$.
11. $\sin x = -\frac{3}{\sqrt{13}}, \tan x = -\frac{3}{2}, \cot x = -\frac{2}{3}, \sec x = \frac{\sqrt{13}}{2}, \csc x = -\frac{\sqrt{13}}{3}$
13. $\sin x = -\frac{3}{\sqrt{10}}, \cos x = -\frac{1}{\sqrt{10}}, \tan x = 3, \sec x = -\sqrt{10}, \csc x = -\frac{\sqrt{10}}{3}$.

Review Exercises

1. a. $\frac{\sqrt{2}}{2}$ c. $\frac{1}{\sqrt{3}}$ e. 1 g. $\frac{1}{2}$ i. $\frac{1}{\sqrt{3}}$ k. $-\frac{2}{\sqrt{3}}$
12. a. $-\frac{\sqrt{2}}{2}$ c. 1 e. 1 g. $-\frac{\sqrt{3}}{2}$ i. 2

Chapter 2

Exercises 2.1

1. $\frac{\pi}{6} \in \mathscr{D}$; $\cos\frac{\pi}{6} - \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3} - 1}{2} \neq 1$.
5. $0 \in \mathscr{D}$; $\cos(2 \cdot 0) = \cos 0 = 1$; $2\cos 0 = 2(1) = 2 \neq 1$.
9. $\pi \in \mathscr{D}$; $\sin\frac{\pi}{2} = 1$; $\frac{1}{2}\sin\pi = \frac{1}{2}(0) = 0 \neq 1$.
13. $\frac{\pi}{6} \in \mathscr{D}$; $\dfrac{\tan\frac{\pi}{6} + 1}{\sin\frac{\pi}{6}} = \dfrac{\frac{1}{\sqrt{3}} + 1}{\frac{1}{2}}$

$= 2\left(\frac{1}{\sqrt{3}} + 1\right) = \frac{2}{\sqrt{3}} + 2$; $\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}} \neq \frac{2}{\sqrt{3}} + 2$.

Exercises 2.2

The equations in exercises 9, 22, and 23 are conditional equations.

10. $\cos\theta\cos(-\theta) - \sin\theta\sin(-\theta) = (\cos\theta)(\cos\theta) - (\sin\theta)(-\sin\theta)$
$= \cos^2\theta + \sin^2\theta$
$= 1$.

11. $\tan^2\theta\csc^2\theta - \sec^2\theta\sin^2\theta = \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\sin^2\theta}\right) - \left(\frac{1}{\cos^2\theta}\right)\sin^2\theta$
$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$
$= \sec^2\theta - \tan^2\theta$
$= 1.$

15. $\sec^4 x - \tan^4 x = (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$
$= (1)(\sec^2 x + \tan^2 x)$
$= \sec^2 x + \tan^2 x.$

20. $\cot^3\beta + 1 = (\cot\beta + 1)(\cot^2\beta - \cot\beta + 1)$
$= (\cot\beta + 1)[(\cot^2\beta + 1) - \cot\beta]$
$= (\cot\beta + 1)(\csc^2\beta - \cot\beta).$

31. $(\cos\beta + \sin\beta)^4 = \cos^4\beta + 4\cos^3\beta\sin\beta + 6\cos^2\beta\sin^2\beta + 4\cos\beta\sin^3\beta + \sin^4\beta$
$= \cos^4\beta + 6\cos^2\beta\sin^2\beta + \sin^4\beta + 4\cos^3\beta\sin\beta + 4\cos\beta\sin^3\beta$
$= (\cos^4\beta + \cos^2\beta\sin^2\beta) + 4\cos^2\beta\sin^2\beta + (\cos^2\beta\sin^2\beta + \sin^4\beta) + 4\cos\beta\sin\beta(\cos^2\beta + \sin^2\beta)$
$= \cos^2\beta(\cos^2\beta + \sin^2\beta) + 4\cos^2\beta\sin^2\beta + (\cos^2\beta + \sin^2\beta)\sin^2\beta + 4\cos\beta\sin\beta\,(1)$
$= \cos^2\beta\,(1) + 4\cos^2\beta\sin^2\beta + (1)\sin^2\beta + 4\cos\beta\sin\beta$
$= \cos^2\beta + 4\cos^2\beta\sin^2\beta + \sin^2\beta + 4\cos\beta\sin\beta.$

42. $\tan^2 z + \cot^2 z = \frac{\sin^2 z}{\cos^2 z} + \frac{\cos^2 z}{\sin^2 z}$
$= \frac{(\sin^2 z)(\sin^2 z) + (\cos^2 z)(\cos^2 z)}{(\cos^2 z)(\sin^2 z)}$
$= \frac{\sin^4 z + (1 - \sin^2 z)(1 - \sin^2 z)}{\sin^2 z\cos^2 z}$
$= \frac{\sin^4 z + 1 - 2\sin^2 z + \sin^4 z}{\sin^2 z\cos^2 z}$
$= \frac{2\sin^4 z - 2\sin^2 z + 1}{\sin^2 z\cos^2 z}.$

Exercises 2.3

1. $\frac{\sqrt{6} - \sqrt{2}}{4}$

3. $\frac{\sqrt{6} - \sqrt{2}}{4}$

5. $-\frac{\sqrt{2} + \sqrt{6}}{4}$

7. $\frac{\sqrt{2} + \sqrt{6}}{4}$

16. $\sin 3\phi = \sin(5\phi - 2\phi)$
$= \sin 5\phi\cos 2\phi - \cos 5\phi\sin 2\phi.$

20. $\frac{\cos(\theta + \phi)}{\cos\theta\cos\phi} = \frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi}$
$= \frac{\cos\theta\cos\phi}{\cos\theta\cos\phi} - \frac{\sin\theta\sin\phi}{\cos\theta\cos\phi}$
$= 1 - \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\phi}{\cos\phi}\right)$
$= 1 - \tan\theta\tan\phi.$

25. $\sec(x + y) = \frac{1}{\cos(x + y)}$
$= \frac{1}{\cos x\cos y - \sin x\sin y}$
$= \frac{\frac{1}{\cos x\cos y}}{\frac{\cos x\cos y}{\cos x\cos y} - \frac{\sin x\sin y}{\cos x\cos y}}$
$= \frac{\left(\frac{1}{\cos x}\right)\left(\frac{1}{\cos y}\right)}{1 - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)}$
$= \frac{\sec x\sec y}{1 - \tan x\tan y}$

39. $(\sin x\cos y)^2 - (\cos x\sin y)^2 = \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$
$= \sin^2 x\cos^2 y + \sin^2 x\sin^2 y - \sin^2 x\sin^2 y - \cos^2 x\sin^2 y$
$= \sin^2 x\,(\cos^2 y + \sin^2 y) - (\sin^2 x + \cos^2 x)\sin^2 y$
$= \sin^2 x\,(1) - (1)\sin^2 y$
$= \sin^2 x - \sin^2 y.$

Exercises 2.4

1. $\frac{\sqrt{3}}{2}$

3. $\sqrt{3}$

5. 0

7. -1

11. $\sin 5x = \sin(3x + 2x)$
$= \sin 3x\cos 2x + \cos 3x\sin 2x$
$= (3\sin x - 4\sin^3 x)(1 - 2\sin^2 x) + (4\cos^3 x - 3\cos x)(2\sin x\cos x)$
$= 3\sin x - 6\sin^3 x - 4\sin^3 x + 8\sin^5 x + 8\sin x\cos^4 x - 6\sin x\cos^2 x$
$= 3\sin x - 10\sin^3 x + 8\sin^5 x + 8\sin x(1 - \sin^2 x)^2 - 6\sin x(1 - \sin^2 x)$
$= 3\sin x - 10\sin^3 x + 8\sin^5 x + 8\sin x(1 - 2\sin^2 x + \sin^4 x) - 6\sin x + 6\sin^3 x$
$= 16\sin^5 x - 20\sin^3 x + 5\sin x.$

16. $\frac{2\tan\alpha}{1 + \tan^2\alpha} = \frac{2\tan\alpha}{\sec^2\alpha}$
$= \frac{2\left(\frac{\sin\alpha}{\cos\alpha}\right)}{\frac{1}{\cos^2\alpha}}$
$= 2\left(\frac{\sin\alpha}{\cos\alpha}\right)(\cos^2\alpha)$
$= 2\sin\alpha\cos\alpha$
$= \sin 2\alpha.$

20. $\sin\left(\frac{\pi}{4} - \theta\right) = \sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta$
$= \frac{\sqrt{2}}{2}(\cos\theta - \sin\theta),$

$$1 - 2\sin^2\left(\frac{\pi}{4} - \theta\right) = 1 - 2\left(\frac{1}{2}\right)(\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta)$$
$$= 1 - [(\cos^2\theta + \sin^2\theta) - 2\sin\theta\cos\theta)]$$
$$= 1 - [1 - \sin 2\theta]$$
$$= \sin 2\theta.$$

25. $\sin\theta\sin 3\theta = \sin\theta(3\sin\theta - 4\sin^3\theta)$
$= 3\sin^2\theta - 4\sin^4\theta$
$= 4\sin^2\theta - \sin^2\theta - 4\sin^4\theta$
$= 4\sin^2\theta(1 - \sin^2\theta) - \sin^2\theta$
$= 4\sin^2\theta\cos^2\theta - \sin^2\theta$
$= (2\sin\theta\cos\theta)^2 - \sin^2\theta$
$= \sin^2 2\theta - \sin^2\theta.$

33. $\cot x + \tan x = \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$
$= \dfrac{(\cos x)(\cos x) + (\sin x)(\sin x)}{\sin x\cos x}$
$= \dfrac{\sin^2 x + \cos^2 x}{\sin x\cos x}$
$= \dfrac{1}{\sin x\cos x}$
$= \dfrac{2}{2\sin x\cos x}$
$= \dfrac{2}{\sin 2x}$
$= 2\csc 2x.$

40. $\dfrac{1 + \sin 2x + \cos 2x}{1 + \sin 2x - \cos 2x} = \dfrac{1 + 2\sin x\cos x + (2\cos^2 x - 1)}{1 + 2\sin x\cos x - (1 - 2\sin^2 x)}$
$= \dfrac{2\sin x\cos x + 2\cos^2 x}{2\sin x\cos x + 2\sin^2 x}$
$= \dfrac{2\cos x(\sin x + \cos x)}{2\sin x(\cos x + \sin x)}$
$= \dfrac{\cos x}{\sin x} = \cot x.$

Exercises 2.5

1. $\dfrac{\sqrt{2 - \sqrt{3}}}{2}$
3. $\dfrac{\sqrt{2 + \sqrt{2}}}{2}$
5. $\dfrac{\sqrt{2 + \sqrt{2 + \sqrt{3}}}}{2}$
7. $\dfrac{\sqrt{2 + \sqrt{2 + \sqrt{2}}}}{2}$
9. $\sqrt{\dfrac{2 - \sqrt{3}}{2 + \sqrt{3}}}$ or $\sqrt{7 - 4\sqrt{3}}$
11. $\sqrt{\dfrac{2 + \sqrt{3}}{2 - \sqrt{3}}}$ or $\sqrt{7 + 4\sqrt{3}}$
13. $\dfrac{\sqrt{2 - \sqrt{2}}}{2}$
15. $\dfrac{\sqrt{2 - \sqrt{2}}}{2}$
17. $\dfrac{\sqrt{2 - \sqrt{3}}}{2}$
19. $\sqrt{\dfrac{2 - \sqrt{2}}{2 + \sqrt{2}}}$ or $\sqrt{3 - 2\sqrt{2}}$

23. $\tan^2\dfrac{\beta}{2} = \dfrac{1 - \cos\beta}{1 + \cos\beta}$
$= \dfrac{(1 - \cos\beta)(1 - \cos\beta)}{(1 + \cos\beta)(1 - \cos\beta)}$
$= \dfrac{(1 - \cos\beta)^2}{1 - \cos^2\beta}$
$= \dfrac{(1 - \cos\beta)^2}{\sin^2\beta}$
$= \left(\dfrac{1 - \cos\beta}{\sin\beta}\right)^2$
$\tan\dfrac{\beta}{2} = \pm\dfrac{1 - \cos\beta}{\sin\beta}.$

if $\tan\frac{\beta}{2} > 0$, $\frac{\beta}{2}$ is in the 1st or 3rd quadrant. If $\frac{\beta}{2}$ is in the 1st quadrant, $\frac{\beta}{2}$ is between $2n\pi$ and $2n\pi + \frac{\pi}{2}$ for some $n \in J$; β is between $4n\pi$ and $4n\pi + \pi$, which means $\sin\beta > 0$, and $\dfrac{1 - \cos\beta}{\sin\beta} > 0$.

If $\frac{\beta}{2}$ is in the 3rd quadrant, $\frac{\beta}{2}$ is between $2n\pi + \pi$ and $2n\pi + \frac{3\pi}{2}$ for some $n \in J$; β is between $4n\pi + 2\pi$ and $4n\pi + 3\pi$, which means $\sin\beta > 0$ and $\dfrac{1 - \cos\beta}{\sin\beta} > 0$.

If $\tan\frac{\beta}{2} < 0$, $\frac{\beta}{2}$ is in the 2nd quadrant or the 4th quadrant. Show that in either case, $\sin\beta < 0$, so $\dfrac{1 - \cos\beta}{\sin\beta} < 0$.

27. $\tan\dfrac{y}{2} = \dfrac{1 - \cos y}{\sin y}$ (from No. 23)
$= \dfrac{1}{\sin y} - \dfrac{\cos y}{\sin y}$
$= \csc y - \cot y.$

34. $\dfrac{1 + \tan\frac{z}{2}}{1 - \tan\frac{z}{2}} = \dfrac{1 + \dfrac{\sin z}{1 + \cos z}}{1 - \dfrac{\sin z}{1 + \cos z}}$
$= \dfrac{\left(\dfrac{1 + \cos z + \sin z}{1 + \cos z}\right)}{\dfrac{1 + \cos z - \sin z}{1 + \cos z}}$
$= \dfrac{1 + \cos z + \sin z}{(1 + \cos z) - \sin z}$
$= \dfrac{1 + \cos z + \sin z}{(1 + \cos z) - \sin z} \cdot \dfrac{1 + \cos z + \sin z}{1 + \cos z + \sin z}$
$= \dfrac{1 + \cos^2 z + \sin^2 z + 2\cos z + 2\sin z + 2\sin z\cos z}{1 + 2\cos z + \cos^2 z - \sin^2 z}$
$= \dfrac{2 + 2\cos z + 2\sin z + 2\sin z\cos z}{1 + 2\cos z + \cos^2 z - (1 - \cos^2 z)}$
$= \dfrac{2[1 + \cos z + \sin z + \sin z\cos z]}{2\cos z + 2\cos^2 z}$
$= \dfrac{2(1 + \cos z)(1 + \sin z)}{2\cos z(1 + \cos z)}$

$$= \frac{1 + \sin z}{\cos z}$$
$$= \frac{1}{\cos z} + \frac{\sin z}{\cos z} = \sec z + \tan z.$$

38. $2\sin^2\frac{y}{6} - \sin^2\frac{y}{7} = 2\left(\frac{1 - \cos\frac{y}{3}}{2}\right) - \left(1 - \cos^2\frac{y}{7}\right)$
$$= 1 - \cos\frac{y}{3} - 1 + \cos^2\frac{y}{7}$$
$$= \cos^2\frac{y}{7} - \cos\frac{y}{3}$$

Exercises 2.6

1. $\frac{1}{4}$ 3. $\frac{\sqrt{2}}{4}$ 5. $-\frac{\sqrt{2}}{4}$

7. $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\cdot\frac{1}{2}\left[\cos\left(\frac{x+y}{2} + \frac{x-y}{2}\right) + \cos\left(\frac{x+y}{2} - \frac{x-y}{2}\right)\right]$
$$= \cos\left(\frac{2x}{2}\right) + \cos\left(\frac{2y}{2}\right)$$
$$= \cos x + \cos y.$$

12. $2\cos 2\alpha \sin 4\alpha = 2\cdot\frac{1}{2}[\sin(4\alpha + 2\alpha) + \sin(4\alpha - 2\alpha)]$
$$= \sin 6\alpha + \sin 2\alpha.$$

17. $\cos 2\theta\,[2\cos 4\theta - 1] = 2\cos 2\theta\cos 4\theta - \cos 2\theta$
$$= 2\cdot\frac{1}{2}[\cos(2\theta + 4\theta) + \cos(2\theta - 4\theta)] - \cos 2\theta$$
$$= \cos 6\theta + \cos(-2\theta) - \cos 2\theta$$
$$= \cos 6\theta + \cos 2\theta - \cos 2\theta$$
$$= \cos 6\theta.$$

19. $2\sin 2x\cos 2x = 2(2\sin x\cos x)(1 - 2\sin^2 x)$
$$= 4\cos x(\sin x - 2\sin^3 x).$$

24. $4\sin x\sin 2x\sin 3x = 2(2\sin x\sin 2x)\sin 3x$
$$= 2\left\{2\cdot\frac{1}{2}[\cos(x - 2x) - \cos(x + 2x)]\right\}\sin 3x$$
$$= 2[\cos(-x) - \cos 3x]\sin 3x$$
$$= 2\sin 3x[\cos x - \cos 3x]$$
$$= 2\sin 3x\cos x - 2\sin 3x\cos 3x$$
$$= 2\cdot\frac{1}{2}[\sin(3x + x) + \sin(3x - x)] - \sin 6x$$
$$= \sin 4x + \sin 2x - \sin 6x.$$

Chapter 3

Exercises 3.1

1. $S = \{x \mid x = -.73 + 2n\pi, n \in J\} \cup \{x \mid x = \pi + .73 + 2n\pi, n \in J\}$.
3. $S = \{z \mid z = -1.107 + n\pi, n \in J\}$.

5. $S = \{\alpha \mid \alpha = \pi - 1.369 + 2n\pi, n \in J\} \cup \{\alpha \mid \alpha = \pi + 1.369 + 2n\pi, n \in J\}$.
7. $S = \emptyset$, $\frac{5}{2}$ is not in the range of the sine function.
9. $S = \{\theta \mid \theta = 1.166 + n\pi, n \in J\}$. 11. $S = \emptyset$.
13. $S = \{z \mid z = .985 + 2n\pi, n \in J\} \cup \{z \mid z = \pi - .985 + 2n\pi, n \in J\}$.
15. $S = \{\alpha \mid \alpha = .464 + n\pi, n \in J\}$. 17. $S = \emptyset$.
19. $S = \{\theta \mid \theta = 2n\pi, n \in J\}$.
21. $S = \{\beta \mid \beta = .896 + n\pi, n \in J\}$.
23. $S = \{y \mid y = 2n\pi, n \in J\} \cup \{y \mid y = 1.231 + 2n\pi, n \in J\} \cup \{y \mid y = -1.231 + 2n\pi, n \in J\}$.
25. $S = \left\{\alpha \mid \alpha = \frac{3\pi}{2} + 2n\pi, n \in J\right\} \cup \{\alpha \mid \alpha = .167 + 2n\pi, n \in J\} \cup \{\alpha \mid \alpha = \pi - .167 + 2n\pi, n \in J\}$.
27. $S = \{\beta \mid \beta = 1.249 + n\pi, n \in J\} \cup \{\beta \mid \beta = -1.249 + n\pi, n \in J\}$.
29. $S = \{t \mid t = .955 + 2n\pi, n \in J\} \cup \{t \mid t = \pi - .955 + 2n\pi, n \in J\} \cup \{t \mid t = \pi + .955 + 2n\pi, n \in J\} \cup \{t \mid t = -.955 + 2n\pi, n \in J\}$.

Exercises 3.2

1. $S = \left\{x \mid x = \frac{\pi}{2} + \frac{2n\pi}{3}, n \in J\right\}$.
3. $S = \left\{\beta \mid \beta = \frac{\pi}{8} + \frac{n\pi}{2}, n \in J\right\}$. 5. $S = \emptyset$.
7. $S = \left\{y \mid y = \frac{\pi}{12} + \frac{n\pi}{2}, n \in J\right\} \cup \left\{y \mid y = -\frac{\pi}{12} + \frac{n\pi}{2}, n \in J\right\}$.
9. $S = \left\{\beta \mid \beta = \frac{\pi}{20} + \frac{n\pi}{5}, n \in J\right\}$.
11. $S = \left\{x \mid x = \frac{\pi}{4} + \frac{n\pi}{2}, n \in J\right\}$.
13. $S = \{\alpha \mid \alpha = -.101 + n\pi, n \in J\} \cup \left\{\alpha \mid \alpha = \frac{\pi}{2} + .101 + n\pi, n \in J\right\}$.
15. $S = \{z \mid z = 3\pi + 4n\pi, n \in J\}$.
17. $S = \{y \mid y = \pi + 2n\pi, n \in J\}$.
19. $S = \{\beta \mid \beta = 2n\pi, n \in J\}$.
21. $S = \{x \mid x = .616 + n\pi, n \in J\} \cup \{x \mid x = -.616 + n\pi, n \in J\}$.
23. $S = \emptyset$.
25. $S = \{z \mid z = 2n\pi, n \in J\} \cup \{z \mid z = .927 + 2n\pi, n \in J\}$.
27. $S = \{y \mid y = \pi - 1.176 + 2n\pi, n \in J\}$.
29. $S = \left\{\beta \mid \beta = \frac{3\pi}{4} + n\pi, n \in J\right\} \cup \{\beta \mid \beta = 1.107 + n\pi, n \in J\}$.
31. $S = \{\alpha \mid \alpha = \pi + 2n\pi, n \in J\}$.
33. $S = \left\{x \mid x = \frac{\pi}{12} + \frac{n\pi}{3}, n \in J\right\} \cup \left\{x \mid x = .369 + \frac{n\pi}{3}, n \in J\right\}$.

Exercises 3.3

1. $S = \left\{\theta \mid \theta = \frac{2n\pi}{7}, n \in J\right\} \cup \{\theta \mid \theta = \pi + 2n\pi, n \in J\}$.
3. $S = \left\{x \mid x = \frac{n\pi}{4}, n \in J, n \neq 4k + 2, k \in J\right\}$.
5. $S = \left\{\beta \mid \beta = \frac{\pi}{7} + \frac{2n\pi}{7}, n \in J\right\} \cup \{\beta \mid \beta = \pi + 2n\pi, n \in J\}$.

7. $S = \left\{y \mid y = \frac{2n\pi}{3}, n \in J\right\}$.

9. $S = \left\{\alpha \mid \alpha = \frac{n\pi}{3}, n \in J, n \neq 3k, k \in J\right\}$.

11. $S = \left\{z \mid z = -\frac{\pi}{2} + 2n\pi, n \in J\right\} \cup \left\{z \mid z = \frac{3\pi}{14} + \frac{2n\pi}{7}, n \in J\right\}$.

13. $S = \left\{x \mid x = \frac{\pi}{5} + \frac{2n\pi}{5}, n \in J\right\}$. Mistake

15. $S = \left\{\theta \mid \theta = \frac{\pi}{4} + \frac{n\pi}{3}, n \in J\right\}$.

Review Exercises

1. $S = \{x \mid x = \pi + .412 + 2n\pi, n \in J\} \cup \{x \mid x = -.412 + 2n\pi, n \in J\}$.

3. $S = \left\{\alpha \mid \alpha = \frac{\pi}{2} + 2n\pi, n \in J\right\} \cup \{\alpha \mid \alpha = .730 + 2n\pi, n \in J\} \cup$
$\{\alpha \mid \alpha = \pi - .730 + 2n\pi, n \in J\}$.

5. $S = \{x \mid x = 2n\pi, n \in J\} \cup \left\{x \mid x = \frac{\pi}{5} + \frac{2n\pi}{5}, n \in J\right\}$.

7. $S = \left\{\beta \mid \beta = \frac{\pi}{2} + .616 + n\pi, n \in J\right\} \cup \left\{\beta \mid \beta = \frac{\pi}{2} - .616 + n\pi, n \in J\right\}$.

9. $S = \{\alpha \mid \alpha = 1.107 + n\pi, n \in J\}$.

11. $S = \left\{y \mid y = -\frac{\pi}{2} + 2n\pi, n \in J\right\}$.

13. $S = \left\{\gamma \mid \gamma = \frac{\pi}{18} + \frac{n\pi}{3}, n \in J\right\}$.

15. $S = \{x \mid x = 1.326 + n\pi, n \in J\} \cup \{x \mid x = 1.249 + n\pi, n \in J\}$.

17. $S = \left\{\theta \mid \theta = -\frac{\pi}{12} + \frac{n\pi}{3}, n \in J\right\}$.

19. $S = \left\{z \mid z = \frac{2\pi}{3} + 2n\pi, n \in J\right\} \cup \left\{z \mid z = \frac{4\pi}{3} + 2n\pi, n \in J\right\}$.

21. $S = \{x \mid x = 2n\pi, n \in J\}$.

23. $S = \left\{\theta \mid \theta = \frac{2n\pi}{5}, n \in J\right\} \cup \left\{\theta \mid \theta = \frac{2n\pi}{9}, n \in J\right\}$.

25. $S = \left\{x \mid x = .464 - \frac{\pi}{3} - n\pi, n \in J\right\} \cup \left\{x \mid x = \frac{\pi}{6} - .464 + n\pi, n \in J\right\}$.

Chapter 4

Exercises 4.1

1. No.
3. Yes. $h^{-1} = \{(5, 1), (3, -1), (9, 3), (1, -3), (13, 5), (-1, -5)\}$.
5. Yes. $g^{-1} = \left\{(x, y) \mid y = \frac{x + 13}{4}\right\}$.
7. No.
9. Yes. $h^{-1} = \left\{(x, y) \mid y = \frac{1 - x}{3}\right\}$.
11. Yes. $g^{-1} = \left\{(w, z) \mid z = \frac{w - 2}{3}\right\}$.
13. Let $f_1 = \{(x, y) \mid y = x^2 - 1, x \geq 0\}$. $f_1^{-1} = \{(x, y) \mid y = \sqrt{x + 1}\}$.

15. Let $h_1 = \{(w, z) \mid z = w^2 + 6w + 9, w \geq -3\}$. $h_1^{-1} = \{(w, z) \mid z = \sqrt{w} - 3\}$.
17. Let $j_1 = \{(u, v) \mid v = u^2 + 4u + 1, u \geq -2\}$. $j_1^{-1} = \{(u, v) \mid v = \sqrt{u + 3} - 2\}$.
19. Let $g_1 = \left\{(x, z) \mid z = \frac{1}{x^2 - 2x - 2}, x \geq 1\right\}$. $g_1^{-1} = \left\{(x, z) \mid z = 1 + \sqrt{\frac{1 - x}{x}}\right\}$.

Exercises 4.2

1. a. $-\frac{\pi}{2}$ c. $-\frac{\pi}{4}$ e. $-\frac{\pi}{3}$ 2. a. π c. $\frac{2\pi}{3}$ e. $\frac{\pi}{2}$
3. a. $\frac{\pi}{6}$ c. $-\frac{\pi}{3}$ e. $-\frac{\pi}{4}$ 4. a. 0 c. $\frac{7\pi}{6}$ e. $\frac{\pi}{3}$
5. $\frac{\pi}{6}$ 7. $-\frac{1}{2}$ 9. $\frac{5\pi}{6}$ 11. $-\frac{\sqrt{2}}{2}$
13. $\frac{\pi}{2}$ 15. $\frac{\pi}{4}$ 17. $\frac{\sqrt{2}}{2}$ 19. $\frac{1}{2}$

Exercises 4.3

3. $\{0\}$ 5. $\left\{\frac{\sqrt{2}}{2}\right\}$ 7. $\{0.8776\}$ 9. $\{0.3272\}$
11. $\{-0.1987\}$

Review Exercises

1. 0 3. $\frac{1}{\sqrt{3}}$ 5. $-\sqrt{2}$ 7. $-\frac{\pi}{2}$
9. $\{0\}$ 11. $\left\{-\frac{\sqrt{2}}{2}\right\}$
13. Yes. $f^{-1} = \{(x, y) \mid y = \sqrt[3]{x + 1}\}$. 15. No.

Chapter 5

Exercises 5.1

1. 75°38′ 3. −62°27′ 5. 71°37′
7. 450° 9. −810° 11. .2327 radians
13. .5061 radians 15. −.5992 radians 17. $-\frac{3\pi}{2}$ radians
19. $\frac{13\pi}{6}$ radians
21.

0°	30°	45°	60°
0 radians	$\frac{\pi}{6}$ radians	$\frac{\pi}{4}$ radians	$\frac{\pi}{3}$ radians

90°	180°	270°	360°
$\frac{\pi}{2}$ radians	π radians	$\frac{3\pi}{2}$ radians	2π radians

23. II
25. I
27. II
29. I
31. quadrantal angle
33. IV
35. I
37. 381°, 741°, −339°, −699°
39. 455°, 815°, −265°, −625°
41. −553°, −913°, 167°, 527°
43. $-\frac{13\pi}{6}$ radians, $-\frac{25\pi}{6}$ radians, $\frac{11\pi}{6}$ radians, $\frac{23\pi}{6}$ radians
45. $\frac{7\pi}{3}$ radians, $\frac{\pi}{3}$ radians, $-\frac{5\pi}{3}$ radians, $-\frac{11\pi}{3}$ radians

Exercises 5.2

Use Figure 5.13 for 1 and 3.

1. $\tan\alpha = \frac{a}{b} = \left(\frac{a}{c}\right) \div \left(\frac{b}{c}\right) = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin x}{\cos x} = \tan x.$
3. $\csc\alpha = \frac{c}{a} = 1 \div \left(\frac{a}{c}\right) = \frac{1}{\sin\alpha} = \frac{1}{\sin x} = \csc x$
5. $\sin\alpha = \frac{15}{17} = .882 = \cos\beta.$ $\cos\alpha = \frac{8}{17} = .471 = \sin\beta.$
   $\tan\alpha = \frac{15}{8} = 1.875 = \cot\beta.$ $\cot\alpha = \frac{8}{15} = .5333 = \tan\beta.$
   $\sec\alpha = \frac{17}{8} = 2.125 = \csc\beta.$ $\csc\alpha = \frac{17}{15} = 1.133 = \sec\beta.$
7. $\sin\alpha = \frac{5}{13} = .385 = \cos\beta.$ $\cos\alpha = \frac{12}{13} = .923 = \sin\beta.$
   $\tan\alpha = \frac{5}{12} = .417 = \cot\beta.$ $\cot\alpha = \frac{12}{5} = 2.4 = \tan\beta.$
   $\sec\alpha = \frac{13}{12} = 1.083 = \csc\beta.$ $\csc\alpha = \frac{13}{5} = 2.6 = \sec\beta.$
9. $\sin\alpha = \frac{40}{41} = .976 = \cos\beta.$ $\cos\alpha = \frac{9}{41} = .220 = \sin\beta.$
   $\tan\alpha = \frac{40}{9} = 4.444 = \cot\beta.$ $\cot\alpha = \frac{9}{40} = .225 = \tan\beta.$
   $\sec\alpha = \frac{41}{9} = 4.556 = \csc\beta.$ $\csc\alpha = \frac{41}{40} = 1.025 = \sec\alpha.$
11. $\alpha = 45°, \beta = 45°, c = 3\sqrt{2}$ ft.
13. $\alpha = 30°, \beta = 60°, b = 3\sqrt{3}$ yds.
15. $\alpha = 45°34', \beta = 44°26', a = 7.142$ in.
17. $\beta = 63°, b = 9.815$ ft, $c = 11.015$ ft.
19. $\alpha = 51°17', a = 18.63$ yds, $c = 23.985$ yds.
21. $\alpha = 63°, b = 2.548$ in., $c = 5.610$ in.
23. $\beta = 27°33', a = 28.76$ ft, $c = 32.43$ ft.
25. 36.1 ft
27. 94.3 ft, 58°, 32°
29. 7.48 ft, 29°56′
31. 355 ft

Exercises 5.3

3. $\gamma = 64°, a = 20.37$ ft, $c = 25.03$ ft.
5. $\alpha = 86°43', a = 3.282$ ft, $b = 1.223$ ft.
7. $\beta = 28°3', \gamma = 115°57', c = 53.57$ in.

9. $\alpha = 43°18', \beta = 104°14', b = 32.51$ yds or $\alpha = 136°42', \beta = 10°40', b = 6.207$ yds.
11. No solution
13. $\alpha = 130°22', \gamma = 11°48', a = 41.0$ in.
15. a. 210 ft b. 703 ft 17. 110 ft 19. 180 ft

Exercises 5.4

1. $\alpha = 65°40', \beta = 40°20', c = 40.09$ ft.
3. $\alpha = 31°45', \beta = 51°15', c = 105.6$ yds.
5. $\alpha = 62°42', \gamma = 58°48', b = 28.31$ in.
7. $\beta = 25°41', \gamma = 22°29', a = 134.8$ ft.
9. $\beta = 42°10', \gamma = 42°10', a = 63.30$ yds.
11. $\alpha = 50°51', \gamma = 70°43', \beta = 58°26'.$
13. $\alpha = 36°52', \beta = 53°8', \gamma = 90°.$
15. 240 ft 17. 37 ft 19. $32\sqrt{3}$ ft

Exercises 5.5

1. 193.9 sq ft
3. 4036 sq yds
5. 450.8 sq in.
7. 778.6 sq ft
9. 606.7 sq ft
11. 280.2 sq in.
13. 191,500 sq yds
15. 170.9 sq ft or 253.6 sq ft
17. No solution
19. 105.8 sq ft
21. $128\sqrt{3}$ sq ft

Review Exercises

1. $\beta = 70°, a = 218.7$ in., $c = 266.4$ in.
3. $\alpha = 40°32', \beta = 49°28', a = 2\sqrt{66}$ ft $= 16.25$ ft.
5. $\alpha = 64°59', \gamma = 40°38', a = 26.92$ ft.
7. a. 10° c. −828° 9. 414.1 ft
11. a. $\frac{2\pi}{9}$ radians c. $-\frac{9\pi}{10}$ radians
13. $\alpha = 35°26', \beta = 48°11', \gamma = 96°23'$ 15. No solution
17. $b = 46.84$ yds, $\alpha = 26°58', \gamma = 103°42'.$

Chapter 6

Exercises 6.1

1. a. 7, −1, 7 + i c. 2, 5, 2 − 5i e. 0, −4, 4i
   g. 9, −2, 9 + 2i i. $\sqrt{3}, \sqrt{2}, \sqrt{3} - \sqrt{2}i$ k. 3, 0, 3
3. $-2\pi - i$
5. $6i$
7. $4\pi + 7i$
9. $1 + 2i$
11. $(3e^2 + 2b) + 37$
13. $-6 - 17i$
15. $\frac{1}{13} + \frac{5}{13}i$
17. $\frac{c}{c^2+d^2} - \frac{d}{c^2+d^2}$
19. $\frac{88}{26} - \frac{41}{26}i$
21. $39 - 21i$
23. $-\frac{79}{125} - \frac{122}{125}i$
25. $-\frac{60}{289} - \frac{32}{289}i$

Exercises 6.2

1. $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
3. $4\left[\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right]$
5. $4\sqrt{3}\left[\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right]$
7. $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
9. $-\frac{5}{\sqrt{2}}+\frac{5}{\sqrt{2}}i$
11. $2-2\sqrt{3}\,i$
13. $-4\sqrt{2}-4\sqrt{2}\,i$
15. $-3\sqrt{3}+3i$

Exercises 6.3

1. $10\left(\cos\frac{13\pi}{6}+i\sin\frac{13\pi}{6}\right)$
3. $6\sqrt{3}\left(\cos\frac{14\pi}{45}+i\sin\frac{14\pi}{45}\right)$
5. $27\left(\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right)$
7. 1
9. $2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
11. $\frac{1}{4}\left[\cos\left(-\frac{2\pi}{15}\right)+i\sin\left(-\frac{2\pi}{15}\right)\right]$
13. 16
15. $144\left[\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right]$
17. $\frac{1}{9}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

Exercises 6.4

1. $1, -\frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}-\frac{\sqrt{3}}{2}i$
3. $1, \cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}, \cos\frac{4\pi}{7}+i\sin\frac{4\pi}{7}, \cos\frac{6\pi}{7}+i\sin\frac{6\pi}{7},$
   $\cos\frac{8\pi}{7}+i\sin\frac{8\pi}{7}, \cos\frac{10\pi}{7}+i\sin\frac{10\pi}{7}, \cos\frac{12\pi}{7}+i\sin\frac{12\pi}{7}$
5. $\sqrt{2}\left[\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right], \sqrt{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right),$
   $\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$
7. $\sqrt[7]{5}\left(\cos\frac{\pi}{35}+i\sin\frac{\pi}{35}\right), \sqrt[7]{5}\left(\cos\frac{11\pi}{35}+i\sin\frac{11\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{21\pi}{35}+i\sin\frac{21\pi}{35}\right), \sqrt[7]{5}\left(\cos\frac{31\pi}{35}+i\sin\frac{31\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{41\pi}{35}+i\sin\frac{41\pi}{35}\right), \sqrt[7]{5}\left(\cos\frac{51\pi}{35}+i\sin\frac{51\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{61\pi}{35}+i\sin\frac{61\pi}{35}\right)$
9. $\sqrt{2}\left[\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right], \sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right),$
   $\sqrt{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right), \sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right),$
   $\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right), \sqrt{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$
11. $\frac{\sqrt{3}}{2}-\frac{i}{2}, i, -\frac{\sqrt{3}}{2}-\frac{i}{2}$

Review Exercises

1. $3\sqrt{3}+3i$
3. a. $3+14i$ c. -3
5. $1+\sqrt{3}\,i, -2, 1-\sqrt{3}\,i$
6. a. $-1+31i$ c. $-5+10i$ e. $27\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
   g. $\cos\frac{8\pi}{15}+i\sin\frac{8\pi}{15}$
7. $12\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
9. a. $\frac{5}{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$ c. $1+9i$
   e. $-24-3i$ g. $-4+4i$

Appendix

Exercises A.1

1. 3
3. $\frac{1}{2}$
5. 0
7. $\log 291 + \log 243$
9. $\log 291 - \log 243$
11. $\frac{1}{2}\log 31 + 3\log 29$
13. $\frac{1}{3}\log 729 + 4\log 281 - \log 927$
15. $4[2\log 314 + \log 475 - \log 627]$
17. $1.967 \cdot 10^{3}$
19. $8.42 \cdot 10^{-2}$
21. $9.189 \cdot 10^{-5}$

Exercises A.2

1. 2.4639
3. 9.7143 − 10
5. 1.4053
7. 8.6894 − 10
9. 6.0208
11. 5.7912 − 10
13. 4.96
15. 820.8
17. 0.6512
19. 1953
21. 0.001328

Exercises A.3

1. $1.466 \cdot 10^{7}$
3. 4291
5. 29.94
7. $3.760 \cdot 10^{10}$
9. 1.239

Exercises A.4

1. 9.8821 − 10
3. 9.9004 − 10
5. 10.2307 − 10
7. 10.0547 − 10
9. 9.9838 − 10
11. 9.5813 − 10
13. 26°50′
15. 48°20′
17. 4°43′
19. 12°52′
21. 23°56′
23. 37°4′