

Quadratic Equations

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Exercises ^[A-1]

Solve:

~~1.~~ $y^2 - 4y = 21$

~~2.~~ $3x^2 - 20 = 11x$

3. $x^2 = 25$

4. $3x^2 - 6x = 9$

5. $2(x^2 + 5) = 9x$

~~6.~~ $(x + 2)(x - 3) = 14$

7. $6z^2 + z = 77$

8. $(z + 2)(2z + 5) = z + 2$

9. $(3y + 5)(3y - 5) = 119$

10. $2x(x - 1) = 3(x - 1)$

~~11.~~ $9z(z - 1) - 3(z - 1) + 1 = 0$

12. $(x + 2)^2 - 4 = 4(x + 1)$

~~13.~~ $12 + 2(x + 1) + 5x(x + 7) = 0$

14. $(y + 3)(y - 1) - y(y - 2) = 2y^2 - 3$

15. $\frac{7x^2}{3} + \frac{17x}{6} = 1$

16. $4x + \frac{15}{x} = 16$

17. $\frac{1 - z - z^2}{z} = \frac{z}{9}$

18. $16x + 16 = \frac{16x + 4}{x}$

19. $y(y - 4) + \frac{y + 6}{3} = 0$

20. $\frac{7y^2}{16} + 28 = 7y$

21. $2x + \frac{7}{x} = 9$

Exercises ^[A-2]

Solve:

1. $x(x - 1) = 20$

2. $2y^2 - 15y = 8$

3. $p^2 = 2p$

4. $6z^2 = 0$

5. $2y^2 + 12 = 11y$

6. $2y^2 + 8 = 8y$

7. $(2x + 1)(x + 3) = 18$

8. $6y^2 = 30y$

9. $10 = 8y(y - 2)$

10. $(x + 1)^2 + (x + 2)^2 = 25$

11. $160y^2 + 124y = 132$

12. $256t^2 = 256t + 512$

13. $(x - 3)(x - 5) + 2x(x - 1) = 92$

14. $y^2 + 2y = 5\frac{1}{4}$

15. $\frac{z^2}{2} = \frac{21z}{5} + 8$

16. $x + \frac{1}{4x} = 1$

17. $\frac{3(x^2 + 1)}{x} = 5 + \frac{1}{x}$

18. $\frac{15}{2x} - 2x = \frac{22}{3}$

19. $p - 5 + \frac{25}{4p} = 0$

20. $\frac{6x + 2}{7} + \frac{3x + 1}{x} = 0$

Some Factorable Third- and Fourth-Degree Equations^[B]

The following examples show how methods of factoring studied in Chapter 10 can be used in the solution of third- and fourth-degree equations.

Example 1. Solve: $x^3 - 3x^2 - x + 3 = 0$.

Solution:

$$\begin{aligned} x^3 - 3x^2 - x + 3 &= 0 \\ x^2(x - 3) - (x - 3) &= 0 \\ (x^2 - 1)(x - 3) &= 0 \\ (x + 1)(x - 1)(x - 3) &= 0 \\ x &= -1, 1, 3 \end{aligned}$$

The solution set is $\{-1, 1, 3\}$.

Example 2. Solve: $4x^4 - 101x^2 + 25 = 0$.

Solution:

$$\begin{aligned} 4x^4 - 101x^2 + 25 &= 0 \\ (4x^2 - 1)(x^2 - 25) &= 0 \\ (2x + 1)(2x - 1)(x + 5)(x - 5) &= 0 \\ x &= -\frac{1}{2}, \frac{1}{2}, -5, 5 \end{aligned}$$

The solution set is $\{-\frac{1}{2}, \frac{1}{2}, -5, 5\}$.

Both Example 1 and Example 2 should be checked by the student before going on. Each answer is checked independently of the others so that, in the second illustration, for example, there are four separate exercises in evaluation to carry out. In each case, the left-hand member of the equation should have the value 0.

Exercises^[B]

NICE

Solve:

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|--|------------------------------|
| 1. $x^3 - 4x^2 - x + 4 = 0$ | 7. $x^4 - 25x^2 - 60x = 36$ |
| 2. $2x^3 - 5x^2 + 2x = 0$ | 8. $z^2(z + 2) = 4z + 8$ |
| 3. $2y^3 - y^2 - 18y + 9 = 0$ | 9. $2x + 3 = 4x^2(2x + 3)$ |
| 4. $z^4 - 5z^2 + 4 = 0$ | 10. $36x^4 = (5x + 1)^2$ |
| 5. $x^4 + 1 = 2x^2$ | 11. $4y^3 - 25y = 50 - 8y^2$ |
| 6. $6y^3 - 7y^2 + y = 0$ | 12. $7x^2 = 3x(x^2 - 2)$ |

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| 1. 7, -3 | 6. 5, -4 | 11. $\frac{2}{3}, \frac{2}{3}$ | 16. $1\frac{1}{2}, 2\frac{1}{2}$ | 1. 1, -1, 4 | 5. 1, 1, -1, -1 | 9. $-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}$ |
| 2. $-1\frac{1}{3}, 5$ | 7. $-3\frac{2}{3}, 3\frac{1}{2}$ | 12. 2, -2 | 17. $\frac{3}{5}, -1\frac{1}{2}$ | 2. $\frac{1}{2}, 2, 0$ | 6. $\frac{1}{6}, 1, 0$ | 10. $-\frac{1}{3}, -\frac{1}{2}, -\frac{1}{6}, 1$ |
| 3. 5, -6 | 8. -2, -2 | 13. $-\frac{2}{5}, -7$ | 18. $-\frac{1}{2}, \frac{1}{2}$ | 3. 3, -3, $\frac{1}{2}$ | 7. -3, -2, 6, -1 | 11. -2, $\frac{5}{8}, \frac{5}{8}$ |
| 4. 3, -1 | 9. -4, 4 | 14. 2, 0 | 19. 3, $\frac{2}{3}$ | 4. 2, -2, 1 | 8. -2, -2, 2 | 12. 0, $-\frac{6}{5}, 3$ |
| 5. $2\frac{1}{2}, 2$ | 10. $1\frac{1}{2}, 1$ | 15. $\frac{2}{7}, -1\frac{1}{2}$ | 20. 8, 8 | | | |
| | | | 21. $3\frac{1}{2}, 1$ | | | |

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|----------------------|---------------------------------|-----------------------------------|-----------------------------------|
| 1. 5, -4 | 6. 2, 2 | 11. $\frac{3}{5}, -1\frac{3}{8}$ | 16. $\frac{1}{2}, \frac{1}{2}$ |
| 2. $-\frac{1}{2}, 8$ | 7. $1\frac{1}{2}, -5$ | 12. 2, -1 | 17. $\frac{2}{3}, 1$ |
| 3. 2, 0 | 8. 5, 0 | 13. $-3\frac{2}{3}, 7$ | 18. $\frac{5}{6}, -4\frac{1}{2}$ |
| 4. 0, 0 | 9. $-\frac{1}{2}, 2\frac{1}{2}$ | 14. $-3\frac{1}{2}, 1\frac{1}{2}$ | 19. $2\frac{1}{2}, 2\frac{1}{2}$ |
| 5. $1\frac{1}{2}, 4$ | 10. -5, 2 | 15. $-1\frac{5}{8}, 10$ | 20. $-3\frac{1}{3}, -\frac{1}{3}$ |