Exercises [A]

Factor:

1. $2x^2 + x - 3$
2. $z^2 - 2z - 8$
3. $5a^2 - 4a - 1$
4. $5c^2 + 4c - 1$
5. $4y^2 - y - 3$
6. $3x^2 - 7x - 6$
7. $1 + b - 6b^2$
8. $a^2 + 4a - 32$
9. $7z^2 + 19z - 6$
10. $15b^2 - 4b - 4$
11. $8x^2 + 10x - 3$
12. $10 + 3y - y^2$
13. $9x^2 + 3x - 2$
14. $p^2 + pq - 2q^2$
15. $6 - 23z - 4z^2$
16. $10x^2 + xy - 2y^2$
17. $3a^2 - 5ab - 12b^2$
18. $63 - 2a - a^2$
19. $9p^2 - 25pq - 6q^2$
20. $x^2 - xy - 56y^2$
21. $6m^2 + mn - 7n^2$

Exercises—Miscellaneous Trinomials [A–1]

Factor:

1. $a^2 + 9a + 8$
2. $x^2 - 5x - 6$
3. $a^2 - 16a + 55$
4. $z^2 - 14z + 24$
5. $2x^2 - 5x + 2$
6. $2p^2 - 5p - 3$
7. $3b^2 - 11b + 6$
8. $5 - 23x - 10x^2$
9. $3b^2 - 17b - 6$
10. $x^2 + 10xz + 25z^2$
11. $6p^2 + 35p - 6$
12. $6 + 13x + 6x^2$
13. $8x^2 - 14x + 3$
14. $10a^2 + 19a + 6$
15. $2a^2b^2 - ab - 15$
16. $12c^2 + 17c + 6$
17. $6a^2 + ab - 15b^2$
18. $8y^2 + 18yz - 5z^2$
19. $9c^2 + 24cd - 20d^2$
20. $4a^2 - 20ab + 25b^2$
21. $3p^2 + 20pq - 100q^2$
22. $12x^2 + 19xy - 10y^2$
23. $-2x^2 + xy + y^2$
24. $x^2 + 72 - 22x$
25. $-48p^2 + 29p + 15$
26. $12m^2 - 32mn + 21n^2$
27. $-21a^2 + 4ab + 12b^2$

Exercises—Miscellaneous Trinomials [A–2]

Factor:

1. $1 - 2x + x^2$
2. $y^2 + 4y - 21$
3. $p^2 + 4p + 4$
4. $x^2 - 14x + 49$
5. $x^2 + xy - 42y^2$
6. $2x^2 - 5xz + 3z^2$
7. $4p^2 - 4p - 15$
8. $2 - 19a + 35a^2$
9. $3y^2 + 5y - 12$
10. $6q^2 - 5q - 6$
11. $4 - 24d + 35d^2$
12. $25a^2 + 10ab - 3b^2$
13. $34b^2 + 9b - 4$
14. $2b^2 - bc - 21c^2$
15. $10 + 3x - 27x^2$
16. $4y^2 - 11y + 6$
17. $40 + x - 6x^2$
18. $9p^2 + 22p + 8$
19. $25p^2 + 5p - 12$
20. $14x^2 + 61xy - 9y^2$
21. $120y^2 - 2 - y$
22. $16z^2 + 49 - 56z$
23. $12 - 25a^2 + 12a^4$
24. $6z^4 - 11z^2 - 7$
25. $108b^2 + 15bc - 7c^2$
26. $21n^2 + 32mn + 12m^2$
27. $12x^2 - 4xy - 21y^2$

Application to the Simplification of Fractions

The student is again reminded that a fraction can be simplified by dividing numerator and denominator by a common *factor*.

Example 1. Simplify $\frac{x^2+3x+2}{x^2+5x+6}$.

Solution: $\frac{x^2+3x+2}{x^2+5x+6}=\frac{(x+1)(x+2)}{(x+2)(x+3)}=\frac{x+1}{x+3}$

Example 2. Simplify $\frac{a^2-4b^2}{a^2+3ab-10b^2}$.

Solution: $\frac{a^2-4b^2}{a^2+3ab-10b^2}=\frac{(a+2b)(a-2b)}{(a+5b)(a-2b)}=\frac{a+2b}{a+5b}$

Example 3. Simplify $\frac{4c^2+12c}{2c^2+3c-9}$.

Solution: $\frac{4c^2+12c}{2c^2+3c-9}=\frac{4c(c+3)}{(c+3)(2c-3)}=\frac{4c}{2c-3}$

Exercises [A–1]

Simplify:

1. $\frac{y^2+5y+6}{y^2+6y+8}$
2. $\frac{x^2-7x+6}{x-1}$
3. $\frac{3x+4}{9x^2-16}$
4. $\frac{s^2-9s+18}{s^2+3s-18}$
5. $\frac{c^2+8c+16}{c^2-16}$
6. $\frac{a(p+q)}{p^2-q^2}$
7. $\frac{a^2-25}{a^2-12a+35}$
8. $\frac{x^2-3x}{x^2-2x-3}$
9. $\frac{4y^2-6y}{4y^2+8y-21}$
10. $\frac{2a^2-5a-3}{4a^2-8a-5}$
11. $\frac{3y-9}{y^2-5y+6}$
12. $\frac{ab+bc+be}{ar+cr+er}$
13. $\frac{c^2-8c+16}{c^2-9c+20}$
14. $\frac{3x^2-15x}{3x^2-16x+5}$
15. $\frac{8x^2+6xy-5y^2}{16x^2-25y^2}$

Solve for x:

16. $kx+4x=k^2+5k+4$
17. $2c(x-c)+5c=x+2$
18. $p(x-q)+6q^2=2(p^2-qx)$
19. $2a(x-4a)+27=3(x+2a)$

Exercises [A–2]

Simplify:

1. $\frac{2a^2+5a+3}{a^2-2a-3}$
2. $\frac{2z^2+z-6}{z+2}$
3. $\frac{2-b}{4-4b+b^2}$

4. $\dfrac{10-7x+x^2}{x^2-4}$

5. $\dfrac{x^2-9}{x^2-7x+12}$

6. $\dfrac{x^2-y^2}{x^2+5xy+4y^2}$

7. $\dfrac{ax+2a}{3x^2+2x-8}$

8. $\dfrac{2p^2-9p+4}{2p^2-8p}$

9. $\dfrac{2b^2-5b+3}{3b^2-4b+1}$

10. $\dfrac{4x^2+16xy+15y^2}{2x^2+xy-10y^2}$

11. $\dfrac{x^2-y^2}{x^2+2xy-3y^2}$

12. $\dfrac{a^2-7a+12}{a^2-a-12}$

Solve for x:

13. $ax-bx=a^2-3ab+2b^2$

14. $x+4h=3h^2+3hx+1$

15. $b(x-b+5)=2(x+3)$

16. $r(x-r)+7(x+7)=0$

Combinations of the Various Types of Factors

In the following three examples, algebraic expressions are factored by using more than one of the methods already considered.

It is important in factoring any expression to look first for a common factor and if there is one, to deal with it before proceeding further.

Example 1. Factor completely $2ax^2-8ay^2$.

Solution: We note that $2a$ is a common factor, so that

$$2ax^2-8ay^2=2a(x^2-4y^2)$$

Now x^2-4y^2 is the difference of two squares, and may be factored.

Thus
$$\begin{aligned}2ax^2-8ay^2&=2a(x^2-4y^2)\\&=2a(x+2y)(x-2y)\end{aligned}$$

Example 2. Factor completely $3x^4-24x^2-27$.

Solution:
$$\begin{aligned}3x^4-24x^2-27&=3(x^4-8x^2-9)\\&=3(x^2-9)(x^2+1)\\&=3(x+3)(x-3)(x^2+1)\end{aligned}$$

Example 3. Factor completely $36a^2+48ab+16b^2$.

Solution:
$$\begin{aligned}36a^2+48ab+16b^2&=4(9a^2+12ab+4b^2)\\&=4(3a+2b)^2\end{aligned}$$

Exercises [A–1]

Factor completely:

1. x^3-xy^2

2. $16-4a^2$

3. $4xa^2-xb^2$

4. $2mr^2+2mrh$

5. $10a^2-25ab+10b^2$

6. $6x+11x^2-10x^3$

7. $49a^2-16b^2$

8. $2x^2-4xy+2y^2$

9. $3a^2 + 18a + 27$
10. $90 + x - x^2$
11. $3a^2 - 19a + 6$
12. $2p^3 - 6p^2 + 4p$
13. $9a^4 - 3a^2$
14. $a^2 + 7a - 78$
15. $-x^2 + 4x - 4$
16. $500c^2d - 20d^3$
17. $3y^2 - 9y + 6$
18. $75z^2 - 48$
19. $3x^2 - 9x - 6$
20. $9x^4 - 81z^4$
21. $a^3b - 4ab^3$
22. $20b^3 - 22b^2 - 12b$
23. $4x^2 - 8xy + 4y^2$
24. $4x^2 + 16x + 16$
25. $-50y^2 + 80y - 32$
26. $60x^2 + 24x - 3$
27. $a^2 + 2a^3x + a^4x^2$
28. $30x^2 - 67x + 33$
29. $2z^3 - 10z^2 - 12z$
30. $a^6 - 9a^4$
31. $25y^4 + 20y^3 + 4y^2$
32. $3a^2 + 6ab + 9a$
33. $2x^2 + 4x + 6$
34. $16x^2 - 40x + 25$
35. $25x^2 + 20xy + 4y^2$
36. $2x^2 - 8xy - 42y^2$
37. $3a^3 - 9a^2 - 30a$
38. $16x^2 - 16x + 4$
39. $12x^3y - 22x^2y - 20xy$
40. $5c^4 - 40c^2 - 100$
41. $6a^2 + 6a - 12$
42. $28a^3b - 35a^2b^2 - 42ab^3$
43. $50a^2 - 130a + 80$
44. $11a^3 + 22a^2b - 88ab^2$

Simplify:

45. $\dfrac{3x^2 - 3y^2}{x + y}$
46. $\dfrac{a^2 - b^2}{a - b}$
47. $\dfrac{2x^2 + 5x + 3}{4x + 6}$
48. $\dfrac{2z^2 + 8z + 8}{3z + 6}$
49. $\dfrac{x^2 - 9y^2}{2x^2 - 10xy + 12y^2}$
50. $\dfrac{2a^2 - 8}{3a^2 - 15a + 18}$
51. $\dfrac{2a^2 + 2ab - 4b^2}{a^2 + 2ab - 3b^2}$
52. $\dfrac{4x^2 - 64y^2}{x^2 + 3xy - 4y^2}$
53. $\dfrac{10x^2 - 80x + 150}{15x^2 - 15x - 90}$

54. Rearrange $4\pi a^2 - \pi b^2$ in the form of an indicated product.

55. Simplify: $\dfrac{2\pi r^2 + 2\pi rh}{2\pi r}$.

56. Find the value of $\dfrac{x^2 - 8x - 9}{(x + 1)^2}$ when $x = -2$.

Page 348

1. (x + 5)(x + 1)
2. (2x + 1)(x + 1)
3. (x + 3)(x + 1)
4. (3x + 1)(x + 1)
5. (x + 7)(x + 1)
6. (7x + 1)(x + 1)
7. (x - 5)(x - 1)
8. (2x - 1)(x - 1)
9. (x - 3)(x - 1)
10. (3x - 1)(x - 1)
11. (x + 3)(x + 2)
12. (x + 5)(x + 2)
13. (x + 5)(x + 3)
14. (x - 7)(x - 3)
15. (x - 11)(x - 2)
16. (5x + 1)(2x + 1)
17. (11x + 1)(3x + 1)
18. (7x - 1)(3x - 1)
19. (3x - 1)(2x - 1)
20. (3x - 1)(3x - 1)
21. (x + 15)(x + 1)
22. (2x + 3)(x + 1)
23. (2x - 3)(x - 1)
24. (2a - 1)(a - 3)
25. (7x - 1)(x - 1)
26. (b - 6)(b - 1)
27. (x - 5)(x - 2)
28. (3y + 1)(y + 2)
29. (1 + 4p)(1 + 3p)
30. (7x - 2)(x - 1)
31. (7z + 1)(z + 2)
32. (4x - 3)(x - 1)
33. (1 + 6q)(1 + 2q)
34. (2y - 5)(2y - 1)
35. (3 - 5a)(2 - a)
36. (1 + 3x)(1 + 3x)
37. (x + y)(x + 3y)
38. (2a - 5b)(2a - 3b)
39. (3x + 7z)(2x + z)
40. (3s + 4)(2s + 1)
41. (a + 2)(a + 1)
42. (2a - 1)(a - 2)
43. (3a + 2)(a + 1)
44. (5a - 2)(a - 1)
45. (3a + 1)(a + 3)
46. (x - 4)(x - 1)
47. (2x - 7)(x - 1)
48. (3x - 2)(x - 2)

Page 349 (top)

1. (2x + 3)(x - 1)
2. (z - 4)(z + 2)
3. (5a + 1)(a - 1)
4. (5c - 1)(c + 1)
5. (4y + 3)(y - 1)
6. (3x + 2)(x - 3)
7. (1 + 3b)(1 - 2b)
8. (a + 8)(a - 4)
9. (7z - 2)(z + 3)
10. (5b + 2)(3b - 2)
11. (2x + 3)(4x - 1)
12. (5 - y)(2 + y)
13. (3x + 2)(3x - 1)
14. (p + 2q)(p - q)
15. (6 + z)(1 - 4z)
16. (5x - 2y)(2x + y)
17. (3a + 4b)(a - 3b)
18. (9 + a)(7 - a)
19. (9p + 2q)(p - 3q)
20. (x - 8y)(x + 7y)
21. (6m + 7n)(m - n)

Page 349 (middle)

1. (a + 8)(a + 1)
2. (x - 6)(x + 1)
3. (a - 11)(a - 5)
4. (z - 12)(z - 2)
5. (2x - 1)(x - 2)
6. (2p + 1)(p - 3)
7. (3b - 2)(b - 3)
8. (5 + 2x)(1 - 5x)
9. (3b + 1)(b - 6)
10. (x + 5z)(x + 5z)
11. (6p - 1)(p + 6)
12. (3 + 2x)(2 + 3x)
13. (2x - 3)(4x - 1)
14. (5a + 2)(2a + 3)
15. (2ab + 5)(ab - 3)
16. (3c + 2)(4c + 3)
17. (3a + 5b)(2a - 3b)
18. (2y + 5z)(4y - z)
19. (3c + 10d)(3c - 2d)
20. (2a - 5b)(2a - 5b)
21. (3p - 10q)(p + 10q)
22. (12x - 5y)(x + 2y)
23. (2x + y)(-x + y)
24. (x - 18)(x - 4)
25. (-16p + 15)(3p + 1)
26. (6m - 7n)(2m - 3n)
27. (-7a + 6b)(3a + 2b)

Page 349 (bottom)

1. $(1 - x)^2$
2. $(y + 7)(y - 3)$
3. $(p + 2)^2$
4. $(x - 7)^2$
5. $(x + 7y)(x - 6y)$
6. $(2x - 3z)(x - z)$
7. $(2p + 3)(2p - 5)$
8. $(2 - 5a)(1 - 7a)$
9. $(3y - 4)(y + 3)$
10. $(3q + 2)(2q - 3)$
11. $(2 - 5d)(2 - 7d)$
12. $(5a + 3b)(5a - b)$
13. $(17b - 4)(2b + 1)$
14. $(2b - 7c)(b + 3c)$
15. $(5 + 9x)(2 - 3x)$
16. $(4y - 3)(y - 2)$
17. $(8 - 3x)(5 + 2x)$
18. $(9p + 4)(p + 2)$
19. $(5p - 3)(5p + 4)$
20. $(7x - y)(2x + 9y)$
21. $(8y + 1)(15y - 2)$
22. $(4z - 7)^2$
23. $(4 - 3a^2)(3 - 4a^2)$
24. $(3z^2 - 7)(2z^2 + 1)$
25. $(36b - 7c)(3b + c)$
26. $(7n + 6m)(3n + 2m)$
27. $(6x + 7y)(2x - 3y)$

Page 350

1. $\frac{y + 3}{y + 4}$
2. $x - 6$
3. $\frac{1}{3x - 4}$
4. $\frac{s - 6}{s + 6}$
5. $\frac{c + 4}{c - 4}$
6. $\frac{a}{p - q}$
7. $\frac{a + 5}{a - 7}$
8. $\frac{x}{x + 1}$
9. $\frac{2y}{2y + 7}$
10. $\frac{a - 3}{2a - 5}$
11. $\frac{3}{y - 2}$
12. $\frac{b}{r}$
13. $\frac{c - 4}{c - 5}$
14. $\frac{3x}{3x - 1}$
15. $\frac{2x - y}{4x - 5y}$
16. $x = k + 1$
17. $x = c - 2$
18. $x = 2p - 3q$
19. $x = 4a + 9$

Pages 350-351

1. $\frac{2a + 3}{a - 3}$
2. $2z - 3$
3. $\frac{1}{2 - b}$
4. $\frac{x - 5}{x + 2}$
5. $\frac{x + 3}{x - 4}$
6. $\frac{x - y}{x + 4y}$
7. $\frac{a}{3x - 4}$
8. $\frac{2p - 1}{2p}$
9. $\frac{2b - 3}{3b - 1}$
10. $\frac{2x + 3y}{x - 2y}$
11. $\frac{x + y}{x + 3y}$
12. $\frac{a - 3}{a + 3}$
13. $x = a - 2b$
14. $x = 1 - h$
15. $x = b - 3$
16. $x = r - 7$

Pages 351-352

1. $x(x - y)(x + y)$
2. $4(2 - a)(2 + a)$
3. $x(2a - b)(2a + b)$
4. $2\pi r(r + h)$
5. $5(2a - b)(a - 2b)$
6. $x(2 + 5x)(3 - 2x)$
7. $(7a - 4b)(7a + 4b)$
8. $2(x - y)^2$
9. $3(a + 3)^2$
10. $(10 - x)(9 + x)$
11. $(3a - 1)(a - 6)$
12. $2p(p - 2)(p - 1)$
13. $3a^3(3a^2 - 1)$
14. $(a + 13)(a - 6)$
15. $(-x + 2)(x - 2)$
16. $20d(5c + d)(5c - d)$
17. $3(y - 2)(y - 1)$
18. $3(5z + 4)(5z - 4)$
19. $3(x^2 - 3x - 2)$
20. $9(x^2 + 3z^2)(x^2 - 3z^2)$
21. $ab(a + 2b)(a - 2b)$
22. $2b(5b + 2)(2b - 3)$
23. $4(x - y)^2$
24. $4(x + 2)^2$

25. $-2(5y - 4)^2$
26. $3(10x - 1)(2x + 1)$
27. $a^2(1 + ax)^2$
28. $(15x - 11)(2x - 3)$
29. $2z(z - 6)(z + 1)$
30. $a^4(a + 3)(a - 3)$
31. $y^2(5y + 2)^2$
32. $3a(a + 2b + 3)$
33. $2(x^2 + 2x + 3)$
34. $(4x - 5)^2$
35. $(5x + 2y)^2$
36. $2(x - 7y)(x + 3y)$
37. $3a(a - 5)(a + 2)$
38. $4(2x - 1)^2$
39. $2xy(3x + 2)(2x - 5)$
40. $5(c^2 - 10)(c^2 + 2)$
41. $6(a + 2)(a - 1)$
42. $7ab(4a + 3b)(a - 2b)$
43. $10(5a - 8)(a - 1)$
44. $11a(a + 4b)(a - 2b)$
45. $3(x - y)$
46. $a + b$
47. $\frac{x + 1}{2}$
48. $\frac{2(z + 2)}{3}$
49. $\frac{x + 3y}{2(x - 2y)}$
50. $\frac{2(a + 2)}{3(a - 3)}$
51. $\frac{2(a + 2b)}{a + 3b}$
52. $\frac{4(x - 4y)}{x - y}$
53. $\frac{2(x - 5)}{3(x + 2)}$
54. $\pi(2a + b)(2a - b)$
55. $r + h$
56. 11

Pages 353-354

1. $a(x + y)(x - y)$
2. $(3x + 5)(2x + 1)$
3. $2(3a + 5)(2a - 1)$
4. $P^3(1 + R)(1 - R)$
5. $(x - 7)^2$
6. $2(8x^2 + 2x + 1)$
7. $(ab + 4)(ab - 4)$
8. $2(x + 3)(x - 1)$
9. $(3a + 4)(2a - 3)$
10. $P(1 + rt)$
11. $(4x + 7)(3x - 5)$
12. $(3x - 7)(4x + 5)$
13. $(12x + 5)(x + 7)$
14. $(12x + 7)(x + 5)$
15. $(12x - 5)(x + 7)$
16. $(12x + 7)(x - 5)$
17. $(6x + 5)(2x + 7)$
18. $(3x + 5)(4x + 7)$
19. $xy(x + 3y)(x - 3y)$
20. $(1 + x^2)^2$
21. $-(x - 3y)^2$
22. $(5p - 2q)^2$
23. $2(3 + 4a)(3 - 4a)$
24. $(ab + 4)(ab - 4)(a^2b^2 + 16)$
25. $4(3 - 2y)^2$
26. $(1 + 4p^2)^2$
27. $2(2x + y)(2x - 3y)$
28. $(8r - 3)(3r + 1)$
29. $x(x + 2)(x + 1)$
30. $(3b + x)(2b - 3x)$
31. $5z^2(5x + 1)(5x - 1)$
32. $(11x - 2y)(7x + y)$
33. $5(r - 16)(r - 10)$
34. $(6a^2 - 7b^2)(a^2 + 2b^2)$
35. $(4r + 9s)^2$
36. $3(a^2 + 3)(a^2 - 3)$
37. $(x^4 - 8)(x^4 + 8)$
38. $x(5x - 4y)(3x - 4y)$
39. $(3x + 40)(x - 20)$
40. $(3z + 40)(3z - 10)$
41. $4(2x - 3)^2$
42. $3(ab - 5)(ab - 2)$
43. $4r(2r + 5)(2r + 3)$
44. $11a(a - 3b)(a + 2b)$
45. $\frac{x}{x - y}$
46. $\frac{2}{x + 2}$
47. $\frac{x - 2}{x + 2}$
48. $\frac{x}{2(3x - 5)}$
49. $\frac{1}{x + 2y}$
50. $\frac{y - 3}{4}$
51. $\frac{x + 1}{x - 2}$
52. $\frac{3(1 - a)}{5 + a}$
53. 528
54. $-2/3$
55. 8
56. $A = \pi(R + r)(R - r)$
    2398 sq.in.