[06-12-15-8-PEA2B]

Equations in Three Variables

If three linear equations in three variables are independent and consistent, they are satisfied by just one set of values of the three variables. The procedure for finding the common solution of the three equations is an extension of the method used for solving a pair of equations in two variables. The procedure is illustrated in the following Example.

Example. Solve the system of equations:

$$4x + 5y + t = 0 \quad (1)$$
$$8x - y + t = 24 \quad (2)$$
$$3x + 2y + 2t = 1 \quad (3)$$

Solution: Eliminate t from equations (1) and (2) by subtraction.

Equation (2)	$8x - y + t = 24$
Equation (1)	$4x + 5y + t = 0$
Subtract.	$4x - 6y = 24$
	$2x - 3y = 12 \quad (4)$

Equation (4) contains the variables x and y. It is necessary to form a second equation in x and y in order to complete the solution. We therefore proceed to eliminate t from equations (1) and (3).

Multiply equation (1) by 2.	$8x + 10y + 2t = 0$
Equation (3)	$3x + 2y + 2t = 1$
Subtract.	$5x + 8y = -1 \quad (5)$

Equations (4) and (5) are now solved for the required values of x and y.

Multiply equation (4) by 5.	$10x - 15y = 60$
Multiply equation (5) by 2.	$10x + 16y = -2$
Subtract.	$-31y = 62$
	$y = -2$

Substitute -2 for y in equation (4). This gives

$$x = 3.$$

Substitute 3 for x and -2 for y in equation (1), giving

$$12 - 10 + t = 0$$
$$t = -2.$$

The solution of the system of equations is $x = 3$, $y = -2$, $t = -2$.
That is, $(x, y, t) = (3, -2, -2)$.

Exercises [A-1]

Solve and check the following systems of equations:

1. $3x - y + 2z = 4$
   $2x + 3y - z = 14$
   $7x - 4y + 3z = -4$

2. $4t - 2u + 3v = 2$
   $5t - 6u + 2v = -1$
   $3t + 4u - 5v = 7$

3. $x + y = 1$
   $y + z = 2$
   $z + x = 5$

4. $x + 2y + 3z = 1$
   $4x - 4y = 3$
   $4y + 6z = 1$

5. $r = 3(s - t)$
   $t = 4(s - r)$
   $r + t = 2s - 5$

6. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 4$; $\frac{1}{x} + \frac{1}{y} = 2$; $\frac{1}{y} - \frac{1}{z} = 3$

Exercises [A-2]

Solve and check the following systems of equations.

1. $x - 2y + 3z = 2$
   $2x - 3y + z = 1$
   $3x - y + 2z = 9$

2. $3r - 2s + t = 7$
   $2r + s - 3t = 1$
   $r + 2s + 2t = 4$

3. $x + y = 3$
   $y + z = 12$
   $z + x = 7$

4. $x + 2y + 2z = 1$
   $3y + 4z = 1$
   $4x - y = 3$

5. $s = 2(t - u)$
   $t = 8(s - u)$
   $s + t = 3u - 1$

6. $\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = -1$; $\frac{1}{y} + \frac{1}{z} = -1$; $\frac{1}{x} + \frac{1}{y} = 1$

Exercises [B]

1. Show that the following system of equations does not have a unique solution:

$$2x + y = 8$$
$$x + z = 3y - 3$$
$$3x + z = 2y + 5$$

2. Find the values of a, b, c which satisfy the following equations:

$$\frac{a+b}{6} = \frac{3b+4c}{12} = \frac{6c+a}{13} = \frac{a-b+10}{7}.$$

3. Show that the following system of equations does not have a unique solution:

$$y + 2z = 6$$
$$y + z = 4 - x$$
$$y + 3z = 8 + x$$

4. Find the values of a, b, c which satisfy the following equations:

$$\frac{b+c}{3} = \frac{2a+b}{4} = \frac{3c-b}{5} = \frac{2a+b+c}{6}$$

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1. (1, 5, 3)
2. (1, 1, 0)
3. (2, -1, 3)
4. $(\frac{1}{2}, -\frac{1}{4}, \frac{1}{3})$
5. (r, s, t) = (9, 11, 8)
6. $(-\frac{1}{3}, \frac{1}{5}, \frac{1}{2})$

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1. (3, 2, 1)
2. (r, s, t) = (2, 0, 1)
3. (-1, 4, 8)
4. $(\frac{2}{3}, -\frac{1}{3}, \frac{1}{2})$
5. (s, t, u) = (6, 8, 5)
6. $(\frac{1}{4}, -\frac{1}{3}, \frac{1}{2})$

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2. (a,b,c) = (8,4,3)
4. (a,b,c) = (3k,2k,4k), k any constant.