Homework: http://math.mnrt.net/

Name KEY ___ raw ___ scaled ___ percent

Math 11 Trimester 2 Exam 2 (137 Points)

Progressions

■ A. Circle the letter of the correct answer. (5 points each)

[1] The sequence *3, 6, 12, 24, 48, 96, ...* is

a. arithmetic.
b. geometric.
c. <u>neither</u> arithmetic and <u>nor</u> geometric.

[2] The sequence *3, 5, 7, 9, 11, 13, 15, 17, 19, ...* is

a. arithmetic.
b. geometric.
c. <u>neither</u> arithmetic and <u>nor</u> geometric.

[3] The sequence *2, 3, 5, 9, 17, 33, 65, ...* is

a. arithmetic.
b. geometric.
c. <u>neither</u> arithmetic and <u>nor</u> geometric.

■ B. Write the first 4 terms of each sequence whose general term is below. (3 points each)

[1] $a_n = 3 + 2n$ 5, 7, 9, 11

[2] $a_n = 3 \cdot 2^n$ 6, 12, 24, 48

[3] $a_n = n^2$ 1, 4, 9, 16

[4] $a_n = (-1)^n 2^n$ −2, 4, −8, 16

■ **C. Answer the following. (11 points each)**

[1] The last term of the sequence given by $a_n = n^2 - 18n$ is 40. How many terms are there?

$a_n = n^2 - 18n = 40$

$n^2 - 18n - 40 = 0$

$(n - 20)(n + 2) = 0$

$n = 20$

$\therefore$ there are 20 terms.

[2] The sum of the first n terms of a sequence is given by $S_n = 3n^2 + n$. Find S_6.

$S_6 = 3(6)^2 + 6$

$= 3 \cdot 36 + 6$

$= 114$

$\therefore$ $S_6 = 114$

Turn page ⟶ Turn page ⟶ Turn page ⟶ Turn page ⟶

[3] In an arithmetic sequence, the 5^{th} term is *22* and the 10^{th} term is *47*. Find the 16^{th} term.

$a_n = a_1 + (n-1)d$

$$\begin{bmatrix} a_5 = a_1 + 4d = 22 \\ a_{10} = a_1 + 9d = 47 \end{bmatrix}$$

$(a_{10} - a_5) \Rightarrow 5d = 25$

$\boxed{d = 5}$

$a_1 + 20 = 22$

$\boxed{a_1 = 2}$

$a_n = 2 + (n-1)5$

$a_{16} = 2 + 15 \cdot 5$

$= 77$

[4] In an arithmetic sequence, the sum of the first *10* terms is *155* and the sum of the first *15* terms is *190*. Find the 6^{th} term.

$S_{10} = \frac{10}{2}(2a+9d) = 155$
$S_{15} = \frac{15}{2}(2a+14d) = 190$
$\Rightarrow$ $10a + 45d = 155$, $15a + 105d = 190$ $\Rightarrow$ $2a + 9d = 31$, $3a + 21d = 38$

$\Rightarrow$ $6a + 27d = 93$, $6a + 42d = 76$ $\Rightarrow 15d = -17 \Rightarrow \boxed{d = -\frac{17}{15}}$

$\frac{15}{2}\left(2a - \frac{14 \cdot 17}{15}\right) = 190 \Rightarrow 15a - 119 = 190 \Rightarrow 15a = 309 \Rightarrow \boxed{a = \frac{103}{5}}$

$a_6 = \frac{103}{5} + 5\left(-\frac{17}{15}\right) = \frac{103}{5} - \frac{17}{3} = \frac{309}{15} - \frac{85}{15} = \frac{224}{15}$

$\therefore A_6 = \frac{224}{15}$

Turn page ⟶ Turn page ⟶ Turn page ⟶ Turn page ⟶

[5] In a certain geometric sequence, each term is positive, the 3rd term is *18* and the 5th term is 162. Find the 8th term.

$a_3 = ar^2 = 18$ $\Rightarrow$ $\frac{162}{18} = \frac{ar^4}{ar^2} \Rightarrow r^2 = 9 \Rightarrow \boxed{r = 3}$

$a_5 = ar^4 = 162$

and $9a = 18 \Rightarrow \boxed{a = 2}$

$a_n = 2 \cdot 3^{n-1}$

$\boxed{a_8 = 2 \cdot 3^7} = 4374$

[6] Find the least number of terms of the geometric sequence *4, 8, 16, 32, ...* which must be taken for their sum to exceed *500.*

$S_n = a\frac{1-r^n}{1-r}$ • $4, 8, 16, 32, \ldots \Rightarrow a_n = 4 \cdot 2^{n-1}$

$S_n = 4\frac{1-2^n}{1-2} = -4(1-2^n).$

$S_n > 500$

$4\left(\frac{1-2^n}{1-2}\right) > 500$

$-4(1-2^n) > 500$

$4(2^n-1) > 500$

$2^n - 1 > 125$

$2^n > 126 \Rightarrow \boxed{n = 7}$

Finding the first term that exceeds 500 is minus 4 pts. It misses the whole distinction of sequence and series (seq. of partial sums).

Turn page ⟶ Turn page ⟶ Turn page ⟶ Turn page ⟶

[7] Find the general term of the sequence *3, 12, 27, 48, 75, 108, 147, 192, ...*

$\{a_n\}$ 3, 12, 27, 48, 75, 108, ...

$\{b_n\}$ 9, 15, 21, 27, 33...

$\{c_n\}$ 6, 6, 6, 6, ...

$$b_n = 9 + \sum_{k=1}^{n-1} 6 = 9 + 6(n-1) = 9 + 6n - 6 = \boxed{6n+3}$$

$$a_n = 3 + \sum_{k=1}^{n-1} (6k+3) = 3 + \frac{6}{2}\left[(n-1)n\right] + 3(n-1)$$

$$= 3 + 3(n^2 - n) + 3n - 3$$

$$= \cancel{3} + 3n^2 - \cancel{3n} + \cancel{3n} - \cancel{3}$$

$$\boxed{a_n = 3n^2}$$

Turn page ⟶ Turn page ⟶ Turn page ⟶ Turn page ⟶

■ **D. Answer the following. (11 points each)**

[1] The sum of the first n terms of a sequence is given by $S_n = 9n^2 - 4n$. Find the general term of the sequence.

$S_n = 9n^2 - 4n$

$a_n = S_n - S_{n-1}$

$a_n = 9n^2 - 4n - [9(n-1)^2 - 4(n-1)]$

$= 9n^2 - 4n - [9(n^2 - 2n + 1) - 4n + 4]$

$= \cancel{9n^2} - 4n - \cancel{9n^2} + 18n - 9 + \cancel{4n} - 4$

$\boxed{a_n = 18n - 13}$

[2] Given that x, x^2, 1 are three successive terms of an arithmetic sequence and $x \neq 1$, find the number that is the common difference.

$x, x^2, 1.$ A. P. $\Rightarrow$ common dif. so,

$d = x^2 - x = 1 - x^2$

$x^2 - x = 1 - x^2$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$\Rightarrow 2x + 1 = 0$

$\boxed{x = -\frac{1}{2}}$

$d = 1 - x^2$

$= 1 - (-\frac{1}{2})^2$

$= 1 - \frac{1}{4}$

$d = \frac{3}{4}$

Turn page ⟶ Turn page ⟶ Turn page ⟶ Turn page ⟶

[3] If $a_n = (-2)^n \left(\frac{1}{2}\right)^{n-1}$, find the sum of the first n terms when n is an even positive integer.

<u>1ST SOLN</u>

$a_1 = (-2)(1) = -2$

$a_2 = 4 \cdot \frac{1}{2} = 2$

$a_3 = (-8)\left(\frac{1}{4}\right) = -2$

<u>Suggests</u>
Sum of even number of terms is ZERO

<u>Proof</u>

$$a_n = (-2)^n \left(\tfrac{1}{2}\right)^{n-1} = (-1)^n \cdot 2^n \cdot 2^{1-n} = (-1)^n \cdot 2^n \cdot 2 \cdot 2^{-n} = (-1)^n \cdot 2$$

so $a_n = 2(-1)^n$. this is G.P. with $a=2$, $r=(-1)$. SO,

$$S_n = 2\,\frac{1-(-1)^n}{1-(-1)} = 1-(-1)^n.$$

Now, if n is even, then $n = 2k$, $k = 1, 2, 3, 4, \ldots$

thus,

$$S_n = 1-(-1)^{2k}$$

$$= 1-\left[(-1)^2\right]^k$$

$$= 1-1^k$$

$$= 0 \quad \text{for all } k \in \{1,2,3,\ldots\}$$

$\therefore\; a_n = (-2)^n\left(\frac{1}{2}\right)^{n-1} \Rightarrow S_n = 0$ for all even $\underline{n}$.