Homework: http://math.mnrt.net/

Name KEY ___ raw ___ scaled ___ percent

Math 11 Trimester 2 Exam 1 (200 points)

Trigonometry

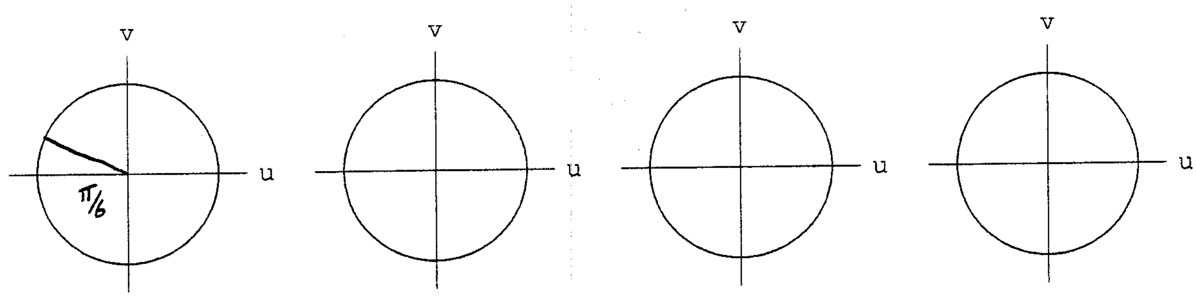
- **Partial credit may be given for correct work. Therefore, it is to your advantage to write clear solutions. If I cannot understand a solution within 90 seconds, then it will receive no partial credit.**

- **Answers must be completely simplified. No denominators may include radicals. All fractions reduced. Arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$.**

- **All angles you write for answers must be written with respect to the angle zero and measured in the positive direction (counter clockwise). For example, write $\theta = \frac{3\pi}{2}$ rather than $\theta = -\frac{\pi}{2}$.**

- **No calculators. All answers must be exact.**

- **All are 10 points.**



A. Find the values of the following.

[1] $\cos \frac{-7\pi}{6} = -\frac{\sqrt{3}}{2}$

[2] $\cos \frac{5\pi}{2} = \cos\left(\frac{4\pi}{2} + \frac{\pi}{2}\right) = \cos\left(2\pi + \frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$

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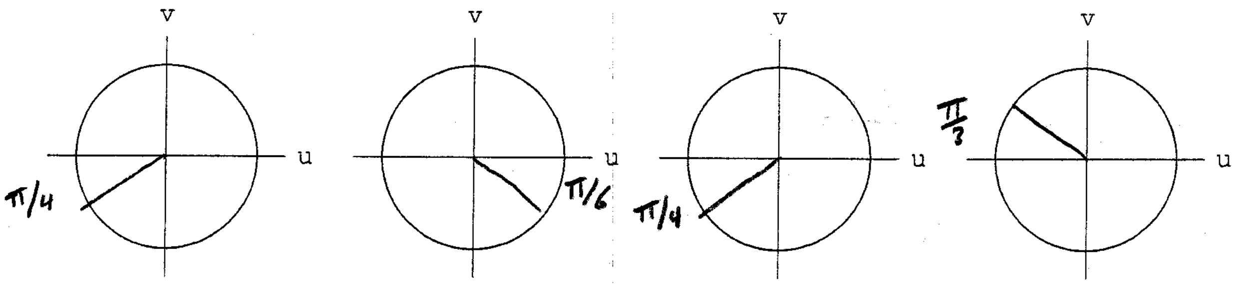
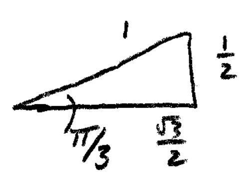


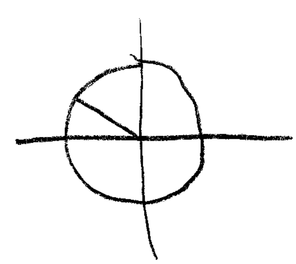
[3] $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

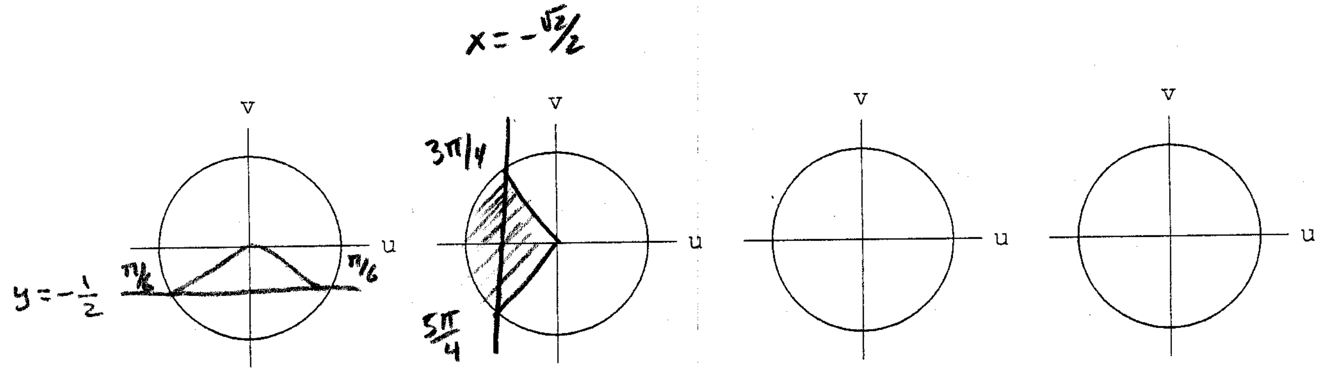
[4] $\sin \frac{11\pi}{6} = -\frac{1}{2}$

[5] $\sin \frac{-3\pi}{4} = -\frac{\sqrt{2}}{2}$

[6] $\tan \frac{-4\pi}{3} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\sqrt{3}$







[7] **Find <u>all</u> values of x for which $\sin x = \frac{-1}{2}$.**

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

$\therefore S = \{x : x = \frac{7\pi}{6} + 2n\pi, n \in \mathbb{Z}\} \cup \{x : x = \frac{11\pi}{6} + 2n\pi, n \in \mathbb{Z}\}$

[8] **Find <u>all</u> values of x, $0 \le x < 2\pi$, for which $\cos x \le \frac{-\sqrt{2}}{2}$**

$\cos x = -\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x = \frac{3\pi}{4} \\ x = \frac{5\pi}{4} \end{cases}$

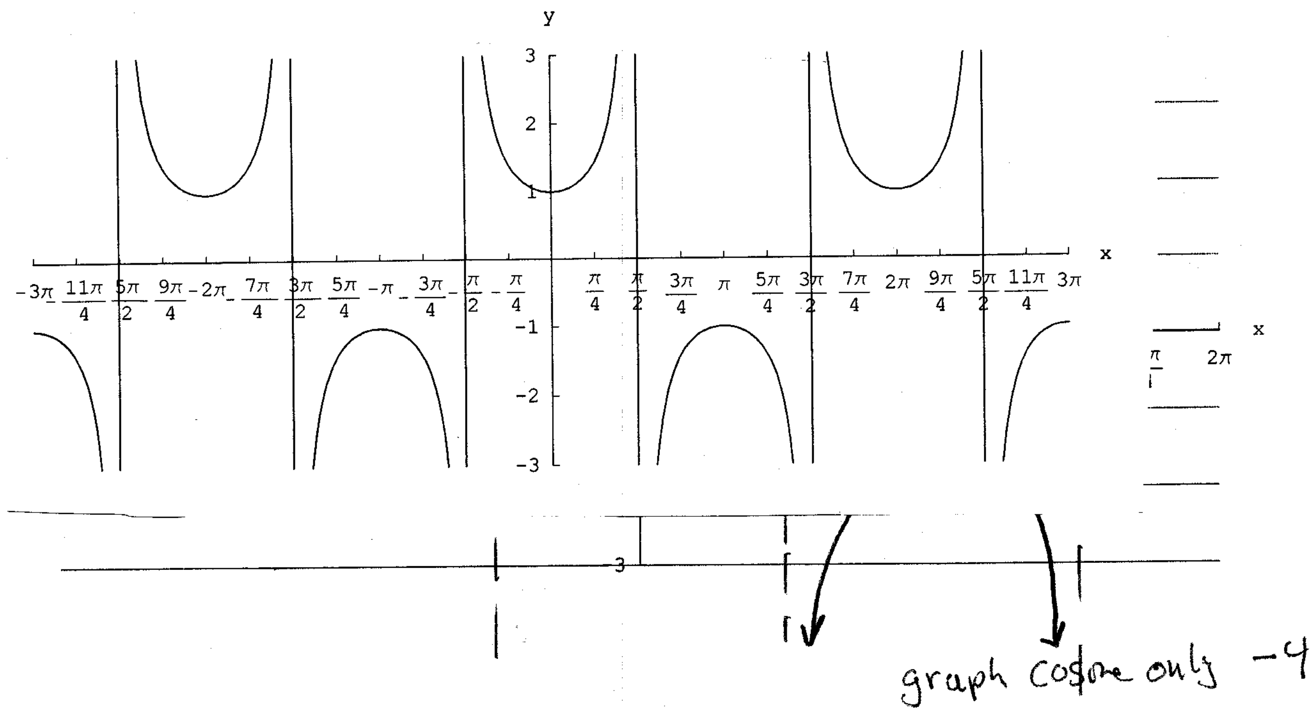
$\frac{3\pi}{4} \le x \le \frac{5\pi}{4}$

$\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$

■ **B1. Graph the following (neatly). Show one full period of the function. Then and answer the questions following the graph.**

[1] The secant function

5pts



State two asymptotes of the secant function.

$2\frac{1}{2}$ pts **1.** $x = -\pi/2$

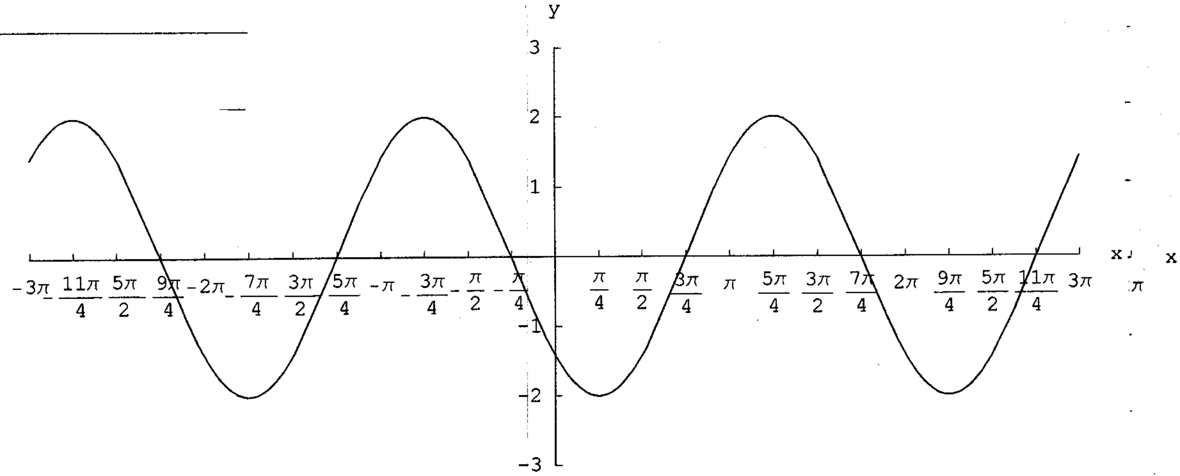
$2\frac{1}{2}$ pts **2.** $x = \pi/2$

$y = \sec x \Rightarrow y = \frac{1}{\cos x} \Rightarrow \cos x \neq 0 \Rightarrow x \neq -\frac{\pi}{2} + n\pi$

[2] $y = 2\sin\left(x - \frac{3\pi}{4}\right)$

4

Amp 1
freq 1
phase 2



State three values of x where the graph of $y = 2\sin\left(x - \frac{3\pi}{4}\right)$ intersects the x-axis.

2 **1.** $x = \frac{3\pi}{4}$

2 **2.** $x = \frac{7\pi}{4}$

2 **3.** $x = \frac{11\pi}{4}$

$\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$

$\frac{3\pi}{4} - \frac{8\pi}{4} = -\frac{3\pi}{4}$

$x - \frac{3\pi}{4} = \frac{\pi}{3}$

$x = \frac{3\pi}{4} + \frac{\pi}{3}$

$= \frac{9\pi}{12} + \frac{4\pi}{12}$

$= \frac{13\pi}{12}$

x	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{11\pi}{4}$	$\frac{9\pi}{4}$	$\frac{19\pi}{4}$
$x - \frac{3\pi}{4}$	0	$\frac{\pi}{2}$	0	$\frac{3\pi}{2}$	0
$\sin(x - \frac{3\pi}{4})$	0	1	0	-1	0
$2\sin(x - \frac{3\pi}{4})$	0	2	0	-2	0

$\frac{4\pi}{3} + \frac{3\pi}{9}$

$\frac{16\pi + 9\pi}{12}$

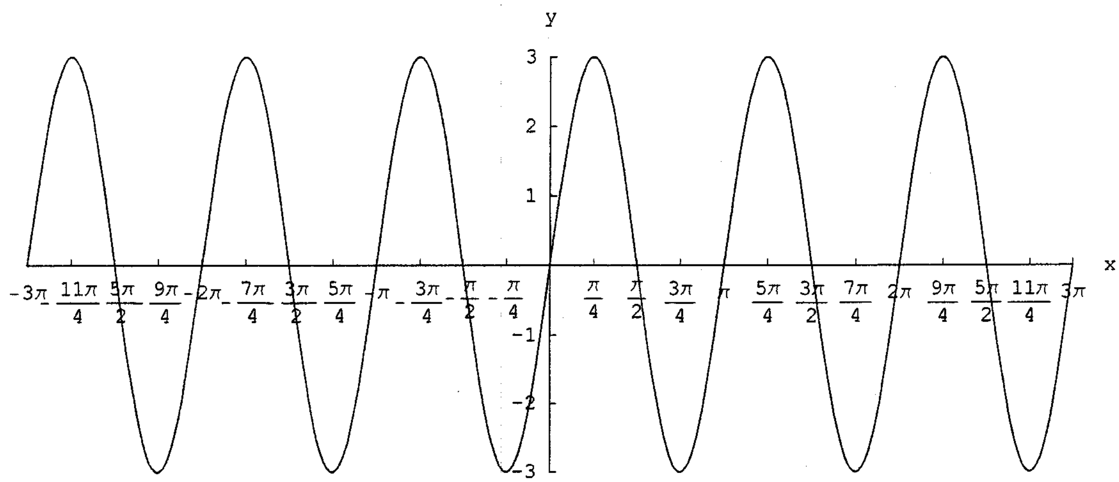
$\frac{25\pi}{12}$

$\frac{3\pi}{4} + \frac{8\pi}{4}$

$\frac{11\pi}{4} + \frac{8\pi}{4} =$

■ **B2. Write the function for each graph.**

[1] The graph crosses the x-axis at ..., $-\pi$, $\frac{-\pi}{2}$, 0, $\frac{\pi}{2}$, π, ...
It has a maximum of 3 and a minimum of -3.



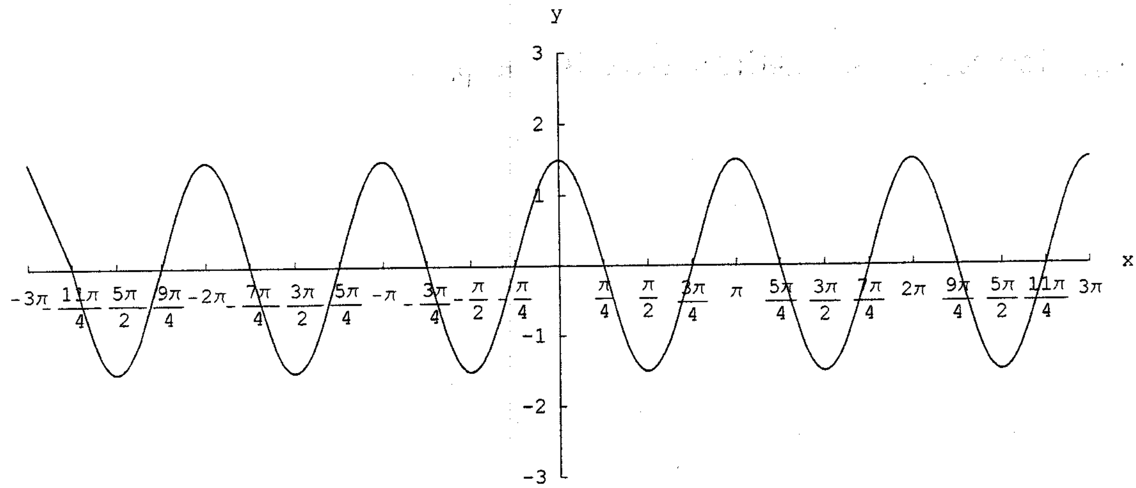
The function of graph [1] is:

Answer	$y = 3\sin 2x$

Ampl freq phase

-0

[2] The graph crosses the x-axis at ..., $\frac{-3\pi}{4}$, $\frac{-\pi}{4}$, $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, ...
It has a maximum of 1.5 and a minimum of -1.5.



The function of graph [2] is:

Answer	$y = 1.5 \sin 2\left(x + \frac{\pi}{4}\right)$

3 3 3

■ **C. Proofs**

[1] Prove any one of the addition theorems that involves only sines and cosines.

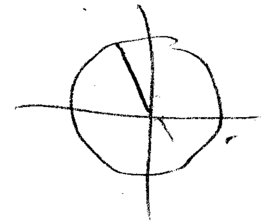
2 pts for knowing an addition formula to prove

[2] Prove: $\cos 2\alpha = 1 - 2\sin^2\alpha$

[3] Prove: $\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$

[4] Prove: $\mathbf{tan\,(x - y) = \frac{tan x - tan y}{1 + tan x\, tan y}}$ (Hint: start with $\tan(x - y) = \frac{\sin(x-y)}{\cos(x-y)}$ and use addition theorems.)

■ **D. Find the following**



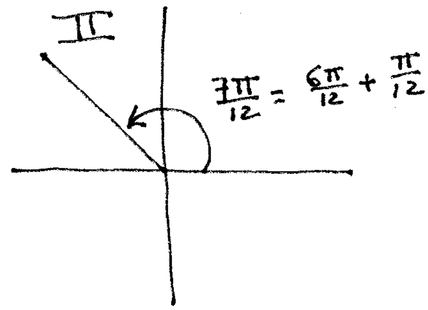
[1] $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi+4\pi}{12}\right) = \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

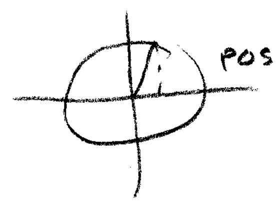
$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$





[2] $\cos\frac{5\pi}{12} = \cos\left(\frac{2\pi+3\pi}{12}\right) = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

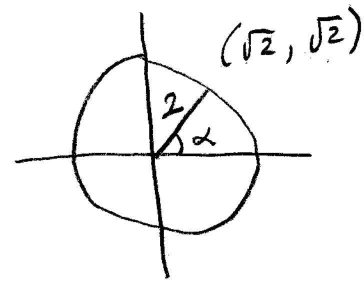
$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\,\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

■ E. Answer the following

[1] Find the maximum and minimum values of $y = \sqrt{2}\sin\theta + \sqrt{2}\cos\theta$ <u>and</u> state the values of θ, $0 \le \theta < 2\pi$, at which they occur.



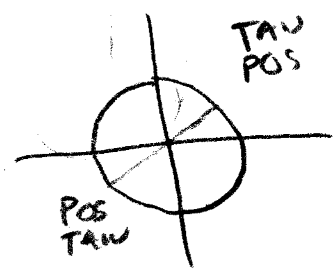
$\sin\alpha = \frac{\sqrt{2}}{2}$, $\cos\alpha = \frac{\sqrt{2}}{2}$, $\alpha = \frac{\pi}{4}$

$$y = \sqrt{2}\sin\theta + \sqrt{2}\cos\theta$$
$$= 2\left[\frac{\sqrt{2}}{2}\sin\theta + \frac{\sqrt{2}}{2}\cos\theta\right]$$
$$= 2(\cos\alpha\sin\theta + \sin\alpha\cos\theta)$$
$$= 2\sin(\alpha+\theta)$$
$$= 2\sin\left(\frac{\pi}{4}+\theta\right)$$

$\therefore$ max $= 2$ when $\theta = \frac{\pi}{4}$

min $= -2$ when $\theta = \frac{7\pi}{4}$

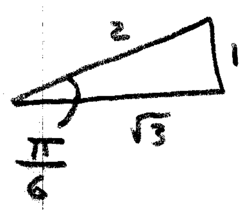
[2] Solve for x, if $13\sqrt{3}\tan 2x - 17 = 12\sqrt{3}\tan 2x - 16$



$13\sqrt{3}\tan 2x - 17 = 12\sqrt{3}\tan 2x - 16$

$\sqrt{3}\tan 2x = 1$ POSITIVE $\Rightarrow$ QUAD I, III

$\tan 2x = \frac{1}{\sqrt{3}}$



$\Rightarrow 2x = \frac{\pi}{6}$

$x = \frac{\pi}{12}$

$\therefore x = \frac{\pi}{12}$, $x = \frac{13\pi}{12}$

$\frac{\pi}{12} + \pi = \frac{13\pi}{12}$