[redacted]

Name KEY ____ raw ____ scaled ____ percent

Math 11 Trimester 1 Exam 3 (307 Points)

BeginningTrig

- **Partial credit may be given for correct work. Therefore, it is to your advantage to write clear solutions. If I cannot understand a solution within 90 seconds, then it will receive no partial credit.**

- **Answers must be completely simplified. No denominators may include radicals. All fractions reduced. Arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$.**

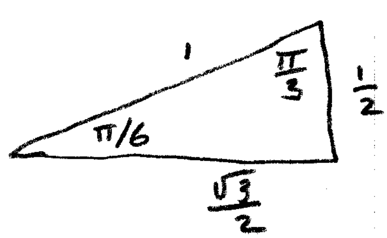
- **All angles you write for answers must be written with respect to the angle zero and measured in the positive direction (counter clockwise). For example, write $\theta = \frac{3\pi}{2}$ rather than $\theta = -\frac{\pi}{2}$.**

- **No calculators. All answers must be exact.**

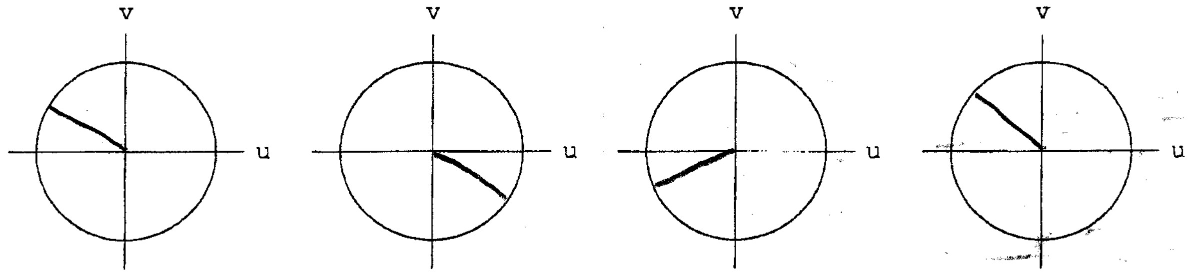
- **A. Fill in the table below. (2 pts each cell)**

[1]

$\theta \longrightarrow$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin(θ)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos(θ)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1



■ **B. Find the following values. (Unit circles provided for your convenience, should you wish to use them.) (19 points each question)**



[1] $\sin \frac{5\pi}{6} = \sin \frac{\pi}{6}$

$= \frac{1}{2}$

[2] $\sin \frac{11\pi}{6} = -\sin(\pi/6)$

$= -\frac{1}{2}$

[3] $\cos \frac{7\pi}{6} = -\cos \frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

[4] $\tan \frac{3\pi}{4} = -\tan \frac{\pi}{4} = -1$

■ **C. Answer the following. (15 points each question)**

[1] If $\tan\theta = \frac{2}{5}$, find the numerical value of $\frac{\sin\theta - 2\cos\theta}{\cos\theta - 3\sin\theta}$

$$\frac{\sin\theta - 2\cos\theta}{\cos\theta - 3\sin\theta} \cdot \frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 2}{1 - 3\tan\theta}$$

$$= \frac{\frac{2}{5} - 2}{1 - 3\left(\frac{2}{5}\right)}$$

$$= \frac{-\frac{8}{5}}{1 - 6/5}$$

$$= \frac{-\frac{8}{5}}{-1/5}$$

$$= \frac{\frac{8}{5}}{\frac{1}{5}}$$

$$= \frac{8}{5} \cdot \frac{5}{1}$$

$$\boxed{= 8}$$

OR

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2}{5}$$

$$\Rightarrow \frac{2 - 10}{5 - 6} = \frac{-8}{-1}$$

$$= 8$$

CHECK

[2] Find the value of $\cos\theta$, if $\sin\theta = -\frac{3}{5}$ and θ is an angle in the third quadrant.

$$\sin^2\theta + \cos^2\theta = 1$$

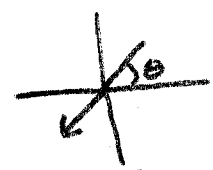
$$\cos^2\theta = 1 - \sin^2\theta$$

$$= 1 - \left(-3/5\right)^2$$

$$= 1 - \frac{9}{25}$$

$$= 16/25$$

$$\cos\theta = 4/5$$



$-\sin\theta$

III QUAD $\Rightarrow$

$$\cos\theta = -\frac{4}{5}$$

■ **D. Prove the following. (15 points each question)**

[1] $\frac{1-\cos\alpha}{1+\cos\alpha} = (\frac{\sin\alpha}{1+\cos\alpha})^2$

$$RHS = \left[\frac{\sin\alpha}{1+\cos\alpha}\right]^2$$

$$= \frac{\sin^2\theta}{(1+\cos\alpha)^2}$$

$$= \frac{1-\cos^2\alpha}{(1+\cos\alpha)^2}$$

$$= \frac{(1+\cos\alpha)(1-\cos\alpha)}{(1+\cos\alpha)^2}$$

$$= \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$= LHS$$

$\square$

OR

$$LHS = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$= \frac{1-\cos\alpha}{1+\cos\alpha}\cdot\frac{1+\cos\alpha}{1+\cos\alpha}$$

$$= \frac{1-\cos^2\alpha}{(1+\cos\alpha)^2}$$

$$= \frac{\sin^2\alpha}{(1+\cos\alpha)^2}$$

$$= \left[\frac{\sin\alpha}{1+\cos\alpha}\right]^2$$

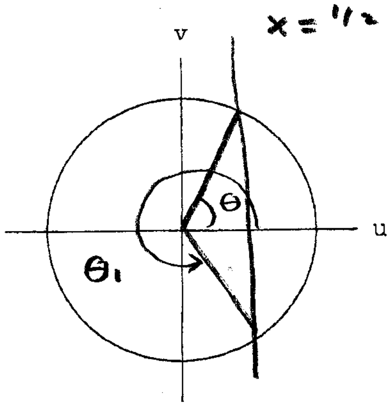
$$= RHS$$

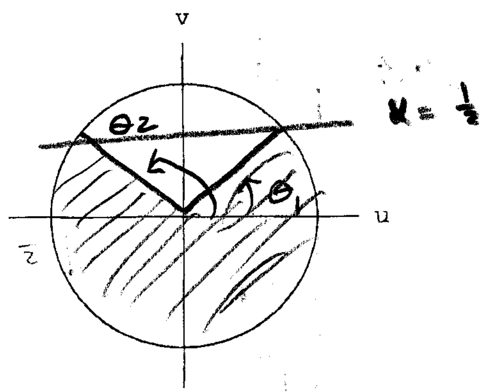
$\square$

use $\sin^2\alpha + \cos^2\alpha = 1$

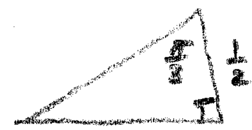
gives at worst -6 pts

■ **E. Answer the following. If your answer requires the use of "and" or of "or" or requires the use of $\cup$ or $\cap$, be sure to use them. (Unit circles provided for your convenience, should you wish to use them.) (23 points each question)**





[1] Find all angles such that $\cos\theta = \frac{1}{2}$.



$\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi/3$

$\theta_1 = 2\pi - \pi/3 = \frac{5\pi}{3}$

$\theta = \frac{5\pi}{3} + 2n\pi$ OR $\theta = \frac{\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$

[2] Find the range of angle θ which satisfies the inequality $\sin\theta < \frac{1}{2}$. Assume that $0 \le \theta < 2\pi$.

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\Rightarrow \boxed{0 \le \theta < \frac{\pi}{6} \text{ or } \frac{5\pi}{6} < \theta \le 2\pi}$

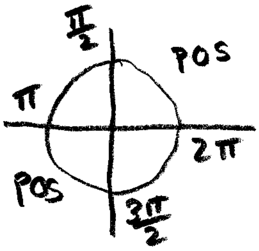
ALT

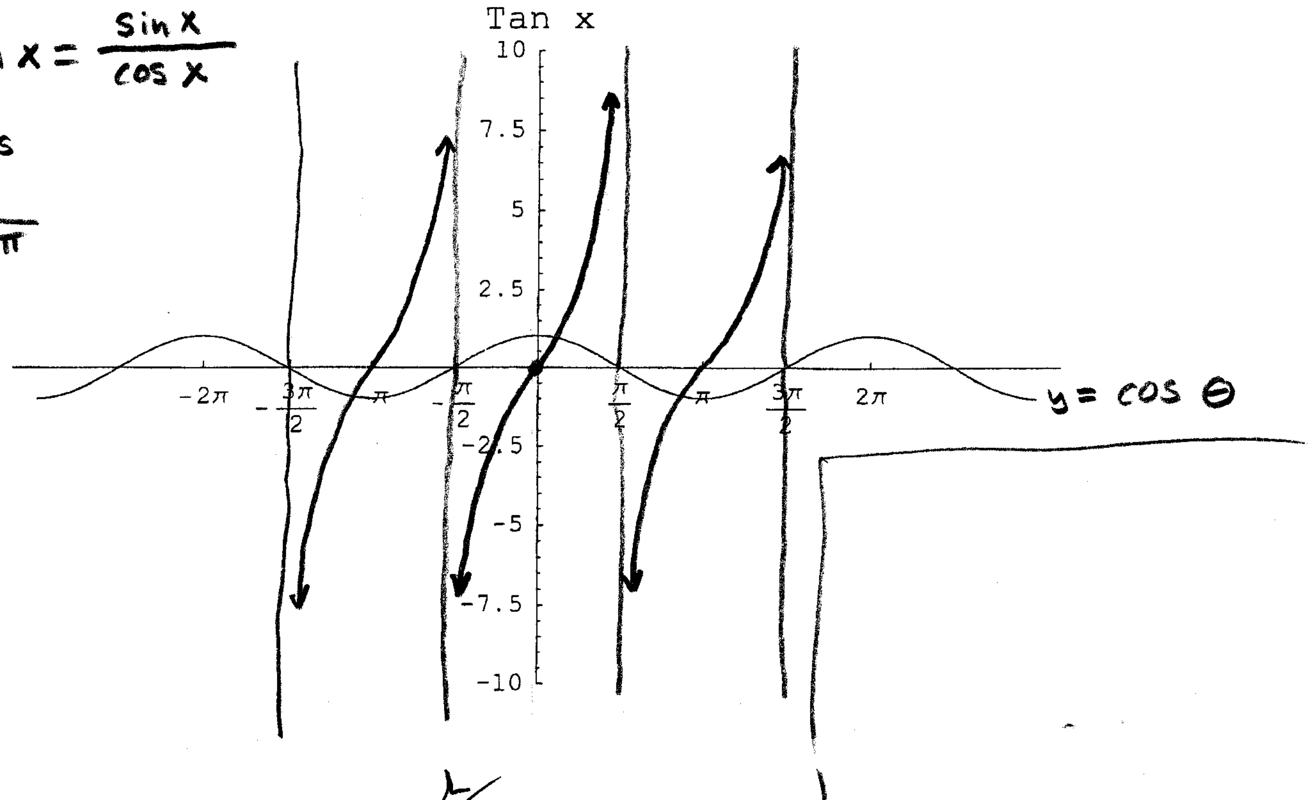
$\theta \in \left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6} \le 2\pi\right]$

■ **F. Graph the following. (31 points each question)**

[1] $y = \tan x,\ x \in \mathbb{R}$

$\tan x = \dfrac{\sin x}{\cos x}$





see back

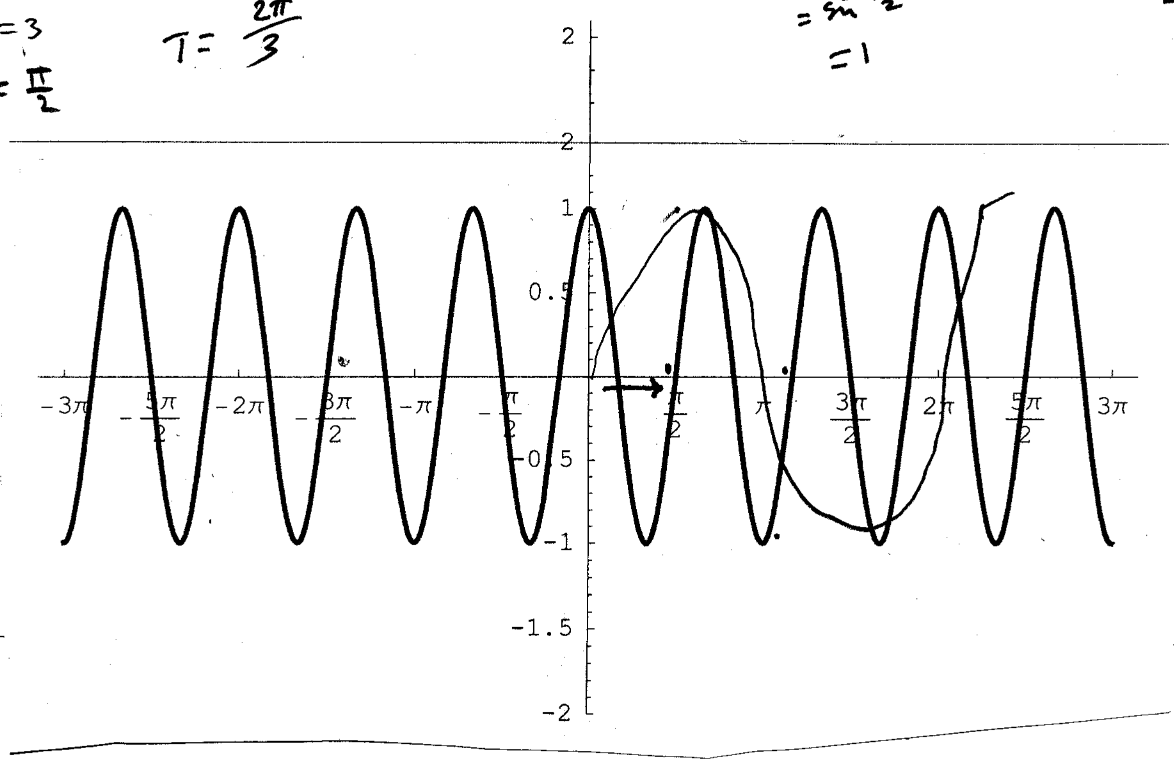
[2] $y = \sin 3(x - \frac{\pi}{2}),\ x \in \mathbb{R}$

$a = 1$
$k = 3$
$\beta = \frac{\pi}{2}$

$T = \frac{2\pi}{3}$

$x = 0$
$\Rightarrow y = \sin 3(-\frac{\pi}{2})$
$= \sin(-\frac{3\pi}{2})$
$=$
$= \sin \frac{\pi}{2}$
$= 1$

CHECK



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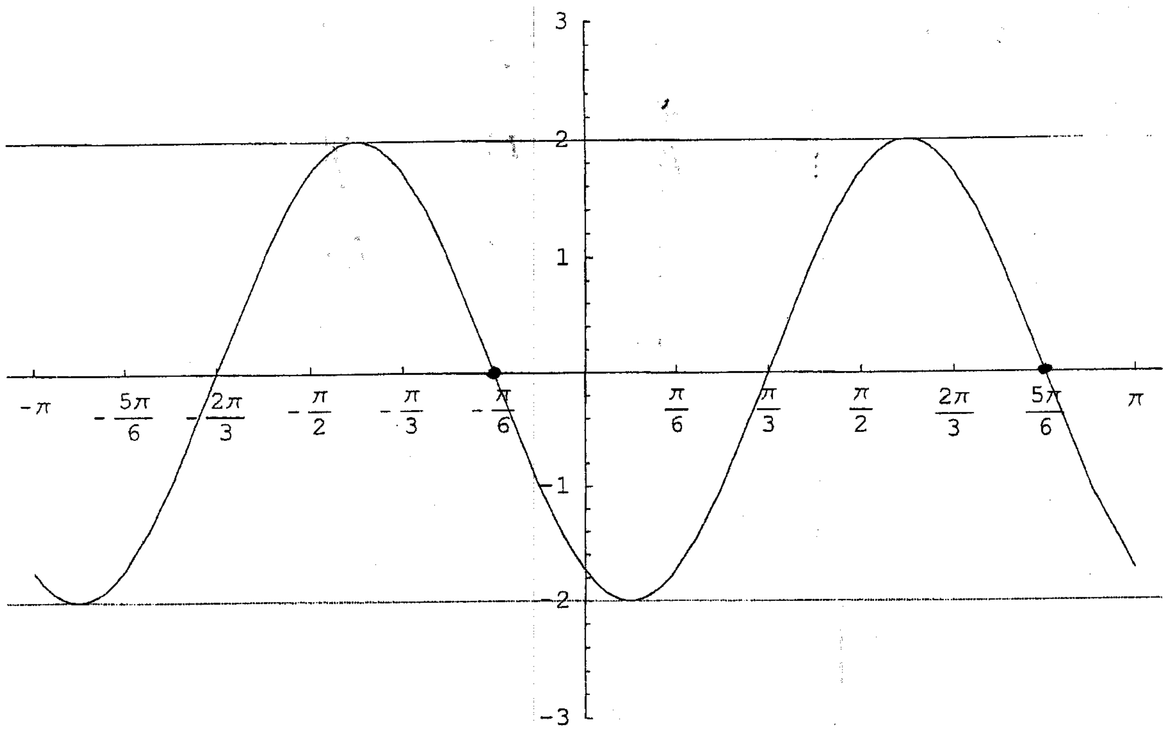
■ G. Answer the following. (23 points each question)

[1] Write the function that corresponds to the following graph.



$a = 2$

$T = \pi$

$k = 2$

$\beta = \frac{\pi}{3}$

$$y = 2 \sin 2\left(x - \frac{\pi}{3}\right)$$

4 items to check

$\sin 3(x - \frac{\pi}{2})$

x	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$
$(x - \pi/2)$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{2\pi}{6}(\frac{\pi}{2})$	$\frac{2\pi}{3}$
$3(x - \frac{\pi}{2})$	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
$\sin 3(x - \frac{\pi}{2})$	0	1	0	-1	0

$x - \frac{1}{2} = \frac{1}{6}$

$\frac{1}{6} + \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$

$\frac{1}{3} + \frac{1}{2}$ $\frac{2}{6} + \frac{3}{6}$ $\frac{2}{6} + \cancel{\frac{1}{2}}\ \frac{8}{6}$ $2 11$

$\frac{4}{6}$ $\frac{3}{6}$ $\cancel{3}$ $\frac{2}{3} + \frac{1}{2}$ $\frac{10\pi}{6}$

$T = \frac{2\pi}{3}$