http://math.mnrt.net/

Name KEY ___ raw ___ scaled ___ percent

Math 11 Trimester 1 Exam 1 (319 Points)
N^{th}roots, functions, exponential equations

- **Partial credit may be given for correct work. Therefore, it is to your advantage to write clear solutions. If I cannot understand a solution within 90 seconds, then it will receive no partial credit.**
- **Answers must be completely simplified. No denominators may include radicals. All fractions reduced. Arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$.**
- **No calculators. All answers must be exact.**

A. Simplify the following. (5 points each answer)

[1] $\sqrt[5]{4}\sqrt[5]{8} = \sqrt[5]{32} = 2$

[2] $\sqrt[3]{27^2} = 3^2 = 9$

[3] $\sqrt[5]{2}\sqrt[3]{2}\sqrt{2} = \boxed{2^{\frac{31}{30}}}$ or $\boxed{\sqrt[30]{2^{31}}}$

or $2^{\frac{30}{30}} \cdot 2^{1/30}$

$= \boxed{2\sqrt[30]{2}}$

$\frac{1}{5} + \frac{1}{3} + \frac{1}{2}$

$\frac{6}{30} + \frac{10}{30} + \frac{15}{30} = \frac{31}{30}$

$\frac{1}{5} + \frac{1}{3} + \frac{1}{2}$

■ **B. Simplify the following. (5 points each answer)**

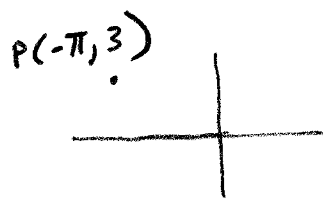
[1] $\left(a^{\frac{5}{12}} a^{\frac{1}{2}}\right)^{\frac{12}{11}}$ $= \left[a^{11/12}\right]^{12/11}$

$= a$

[2] $\left(a^{\sqrt{2}}\right)^{-\pi} \cdot a^{\sqrt{2\pi^2}}$ $= a^{-\pi\sqrt{2}} \cdot a^{\pi\sqrt{2}} = a^{0} = 1$

[3] $\left(5^{\frac{1}{2}} \cdot 5^{-1}\right)^{-2} \cdot 5^{\frac{-3}{2}}$ $= \left[5^{-\frac{1}{2}}\right]^{-2} \cdot 5^{\frac{-3}{2}}$

$= 5^{1} \cdot 5^{-3/2} = 5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$

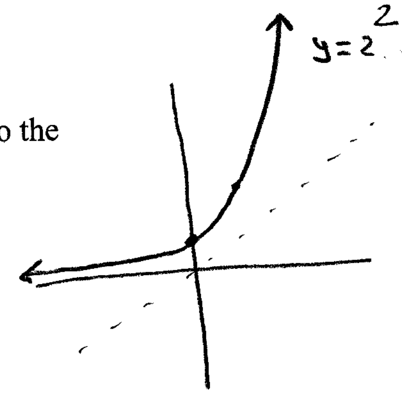


■ **C. Answer the following. (6 points each answer)**

[1] Suppose $P(-\pi, 3)$ and Q are symmetric with respect to the x-axis. Then Q is the point ($-\pi$, -3)

[2] Suppose $P(-3, -9)$ and Q are symmetric with respect to the y-axis. Then Q is the point (3, -9)

[3] Let $P(\pi, 7)$ and Q be symmetric with respect to the line $y = x$. Then Q is the point (7, π)



[4] (Circle the best answer.) The graph of $f(x) = 2^x$ is symmetric with respect to the

(a) the x-axis.

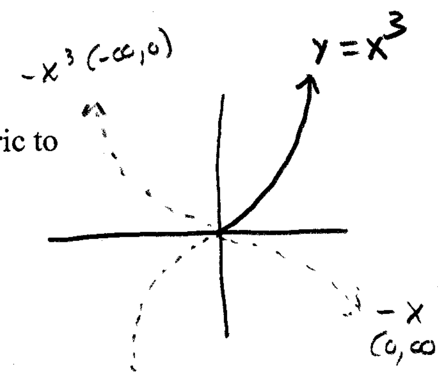
(b) the y-axis.

(c) the line $y = x$.

(e) a, b, and c

(f) a and b only

(g) none of the above.



[5] (Circle the best answer.) The graph of $f(x) = x^3$, $x \in (0, \infty +)$ is symmetric to

get

+ 2pt Y (a) $g(x) = -x^3$, $x \in (0, \infty +)$ with respect to the x-axis.

+ 2pt Y (b) $h(x) = -x^3$, $x \in (-\infty, 0)$ with respect to the y-axis.

+ 2pt Y (c) $j(x) = x^3$, $\in (-\infty, 0)$, with respect to the origin.

5

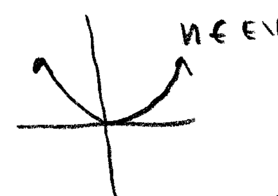
−0 (d) all of the above

Give 2 pts For each correct

−2 (e) a and c only

−6 (f) none of the above.

■ D1. True or False. Check [T] or [F]. (13 points each answer)



[1] If n is an even number, $f : f(x) = x^n$ is an increasing function of x on the interval

[T] [F] $(-\infty, 0)$

☆ BAD Q [T] [F] $(0, \infty +)$

→ [T] [F] $(-\infty, \infty +)$ or [T] may be ok, It is increasing some where.

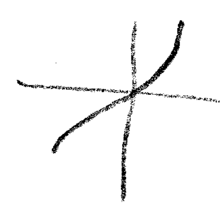
Accept Either

[2] If n is an odd number, $f : f(x) = x^n$ is an increasing function of x on the interval

[T] [F] $(-\infty, 0)$

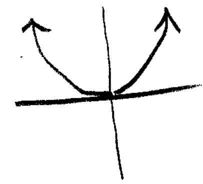
[T] [F] $(0, \infty +)$

[T] [F] $(-\infty, \infty +)$



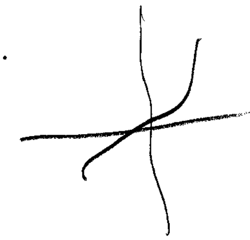
■ D2. Answer the following. (13 points each answer)

[1] If n is an even number, state three points on the graph of $f : f(x) = x^n$.



$(0,0), (1,1), (-1,1)$

[2] If n is an odd number, state the three points on the graph of $f : f(x) = x^n$.



$(0,0), (1,1), (-1,-1)$

[3] Write all solutions of the simultaneous equations $y = x^{16}$ and $y = x^{17}$.

$\{(0,0), (1,1)\}$

■ E. Answer the following. (16 points each answer - (a) and (b) 8 points each)

[1] Let $f : f(x) = ax^2$ and $g : g(x) = bx^2$.

(a) Prove that the composition of f and g is not in general commutative.

Counter example.

Let $a=3, b=2, x=1$

$f(x) = 3(1^2) = 3$

$g(x) = 2(1^2) = 2$

$f(g(x)) = 3(2^2) = 12$

$g(f(x)) = 2(3^2) = 18$

$12 \neq 18 \Rightarrow f \circ g \neq g \circ f$

$a =$

$\frac{a}{b} = \frac{a^2}{b}$

a

(b) For what value(s) (other than zero) of a and b is the composition of f and g commutative.

$f(g(x)) = g(f(x))$

$\Rightarrow ab^2x^4 = a^2bx^4$

$\Rightarrow ab^2 = a^2b$

$\Rightarrow \frac{a}{b} = \frac{a^2}{b^2} \Rightarrow \frac{a}{b} = \left(\frac{a}{b}\right)^2 \Rightarrow$ whenever $a=b$.

$\therefore \{(a,b), a=b\}$

[2] Let $f : f(x) = x^2$ and $g : g(x) = 2x$. Write $g \circ f$ in its simplest form.

$g(f(x)) = 2[x^2] = 2x^2$

[3] For $f : f(x) = x^{-2}, x \neq 0$, evaluate $(f \circ f \circ f)(x)$ at $x = 2$.

[1] $f(f(f(x)))$

$= f(f(x^{-2}))$

$= f((x^{-2})^{-2}$

$= ((x^{-2})^{-2})^{-2}$ $= x^{-8}$

$f(f(f(2))) = \frac{1}{2^8}$

$= \frac{1}{256}$

or

$f(2) = \frac{1}{4}$

$f(\frac{1}{4}) = (\frac{1}{4})^{-2} = 16$

$f(16) = 16^{-2} = \frac{1}{256}$

EASIEST

[4] Let $f : f(x) = x^2 + 2x + 3$ and $g : g(x) = \sqrt{x}$. Evaluate $f(g(x))$ at $x = 9$.

$g(9) = \sqrt{9} = 3$

$f(g(9)) = 3^2 + 2(3) + 3$

$= 9 + 6 + 3$

$= 18$

OR

$f(g(x)) = (\sqrt{x})^2 + 2\sqrt{x} + 3$

$f(g(9)) = 9 + 6 + 3$

$= 18$

[5] Find functions f and g such that $f \circ g = h$, for $h : h(x) = (3x^2 + x - 2)^{\frac{1}{5}}$.

$$\boxed{\begin{aligned} f(x) &= x^{1/5} \\ g(x) &= 3x^2 + x - 2 \end{aligned}}$$

$f(g(x)) = (3x^2 + x - 2)^{1/2}$

[6] State the domain and range of the function f defined by $f(x) = \frac{1}{\sqrt{x-3}}$.

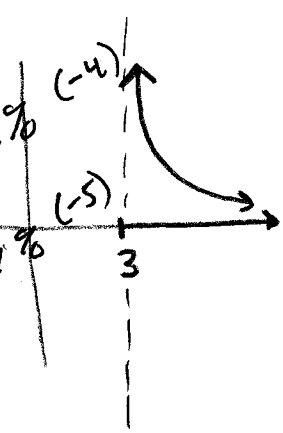
$D_f = x \in \mathbb{R} \ni x > 3$

$R_f = y \in \mathbb{R} \ni y > 0$

~~Rng~~

Dmn ok → get 12 pts = 75%

Rng ok
Dmn wrong → get 11 pts = 69%



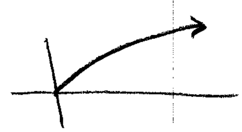
■ **F. Find the inverse of each function and state its domain. (13 points each answer)**

[1] $f : f(x) = 2x - 5$ $\quad D_{f^{-1}} = \mathbb{R}$

$x = 2y - 5$

$f^{-1}(x) = y = \frac{x+5}{2}$

[2] $f : f(x) = \sqrt{x}, x \in [0, \infty)$



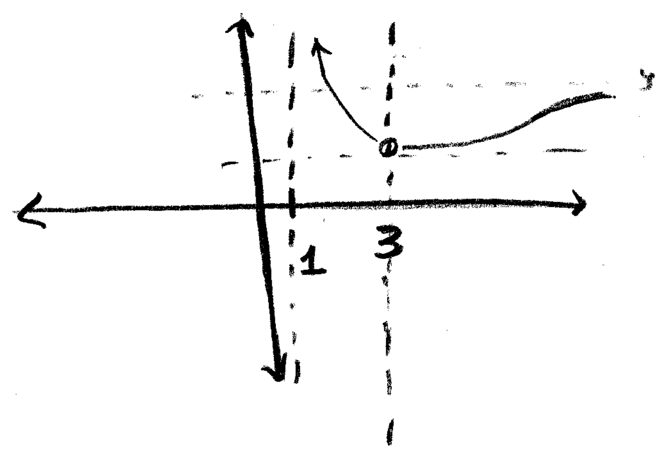
$x = \sqrt{y}$

$x^2 = y$

$f^{-1}(x) = y = x^2$. $\quad D_{f^{-1}} = x \in \mathbb{R} \ni x \geq 0$

$y = \frac{x}{x}\left(\frac{3 - 1/x}{1 - 1/x}\right)$

? good $\quad x \to \infty, y \to 3$



[3] $g : g(x) = \frac{1+x}{3-x}$. $\quad D_f = \mathbb{R} - \{3\}$

$x = \frac{y+1}{3-y}$

$(3-y)x = y+1$

$3x - xy = y + 1$

$y + xy = 3x - 1$

$y(1+x) = 3x - 1$

$f^{-1}(x) = y = \frac{3x-1}{x+1}$, $\quad D_{f^{-1}} = x \in \mathbb{R} \ni x \neq 3$ and $x \neq -1$

■ **G. Solve the following. (11 points each answer)**

[1] $10^{x+1}(100^{2-x}) = \frac{1}{1000}$

$10^{x+1} \cdot 10^{4-2x} = 10^{-3}$

$\therefore \; x+1+4-2x=-3$

$-x+5=-3$

$\boxed{x=8}$

BAD QUESTION

[2] $7^{x^2} = \frac{1}{49^{6-2x}}$

$7^{x^2} = 49^{-6+2x}$

$7^{x^2} = 7^{-12+4x}$

$x^2 = 4x-12$

$x^2-4x+12=0$

$x = \frac{4 \pm \sqrt{16-4(12)}}{2}$

$\boxed{\varnothing}$ in $\mathbb{R}$

$2 \,|\, 32$
$2 \,|\, 16$
$2 \,|\, 8$
$2 \,|\, 4$
2

OR

$x = \frac{4 \pm \sqrt{-32}}{2} = \frac{4 \pm 4i\sqrt{2}}{2}$

$= 2 \pm 2i\sqrt{2}$ in $\mathbb{C}$

[3] $3(2^{x+1}) - 8 = 5(2^x)$

$3(2^{x+1}) - 2^3 = 5(2^x)$

$3[2^x \cdot 2] - 2^3 = 5(2^x)$

$6 \cdot 2^x - 2^3 = 5 \cdot 2^x$

$2^x = 2^3$

$\boxed{x=3}$

■ **H. Answer the following. (13 points each answer)**

[1] Prove that $f : f(x) = x^2 + 2,\ x \geq 2$ is the inverse function of $g : g(x) = \sqrt{x-2},\ x \geq 2$.

Show $f(g(x)) = I(x) = 1$

$$f(g(x)) = (\sqrt{x-2})^2 + 2$$
$$= x - 2 + 2$$
$$= x$$
$$= I(x)$$

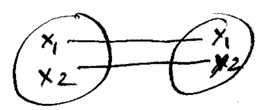
$\square$

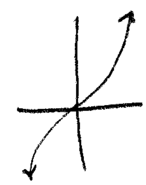
$y_1 \neq y_2 \Rightarrow x_1 \neq x_2$

$x_1 = x_2 \Rightarrow y_1 = y_2$

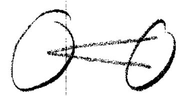
[2] Prove that the function defined by $g : g(x) = x^n,\ n \in \{1, 3, 5, 7, \ldots\}$ is a 1-1 function.

Show $y_1 = y_2 \Rightarrow x_1 = x_2$; $y_1 = g(x_1),\ y_2 = g(x_2)$



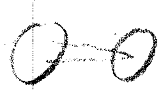


If $y_1 = y_2$



$y_1 \neq y_2 \Rightarrow x_1 \neq x_2$

$x_1 = x_2 \Rightarrow y_1 = y_2$) Function



$x_1 \neq x_2 = y_1 \neq y_2$

$\Rightarrow y_1 = y_2 \Rightarrow x_1 = x_2$ 1-1

$$y_1 = x_1^n$$
$$y_2 = x_2^n$$
$$x_1^n = x_2^n$$
$$\sqrt[n]{x_1} = \sqrt[n]{x_2}$$
$$x_1 = x_2 \quad \because n \text{ odd}$$

$\therefore\ y_1 = y_2 \Rightarrow x_1 = x_2$, g is 1-1 Function.

$\square$