1. Solve the following equations.

(a) $3^{2x} = 27$ (b) $4^x = 32$

(c) $(\sqrt{2})^{3x} = \frac{1}{8}$ (d) $\left(\frac{1}{9}\right)^{x+2} = 3$

(e) $4^x(5^{2x}) = 10$ (f) $5^x - 25 = 0$

(g) $7^{x^2-4} - 1 = 0$ (h) $8^x = 4^{x+1}$

(i) $9^x = (\sqrt{3})^{x+2}$ (j) $\frac{4^x}{2^{x-1}} = 8^{2-x}$

(k) $2^{x^2} = 4^{2(x-1)}$ (l) $7^{x^2} = 49^{6-2x}$

(m) $4^{x^2-6} - 16^{x+1} = 0$ (n) $3^{x^2} = \frac{9^{x+1}}{27^x}$

2. Given that $y = ax^n - 23$, and that $y = 4$ when $x = 3$ and $y = 220$ when $x = 9$, find the value of a and of n.

3. Solve the following simultaneous equations.

(a) $5^x(25^{2y}) = 1$ and $3^{3x}(9^y) = \frac{1}{9}$

(b) $2(4^x) = 32^y$ and $\frac{125^x}{25^y} = 625$

(c) $\frac{3^x}{9^y} = 27$ and $4^{2x}(2^{6y}) = \frac{1}{4}$

4. Show that $\sqrt[n]{2 \times 4^m} = 2^{\frac{2m+1}{n}}$. Hence find the value of m and of n which satisfy the equations $\sqrt[n]{2 \times 4^m} = 8$ and $\frac{27^m}{9^{n+1}} = 81$ simultaneously.

5. By using the substitution $y = 2^x$, find the value of x such that
$$3(2^{x-1}) = 2^x + 4.$$

6. By using the substitution $y = 3^x$, find the values of x such that

(a) $3^x = 4 - 3(3^{-x})$, (b) $3^{2x} - 3^{x+2} = 3^x - 9$.

7. By using appropriate substitutions or otherwise, solve the following equations.

(a) $5^{2x} - 6(5^x) + 5 = 0$

(b) $2^{2x} - 10(2^x) + 16 = 0$

(c) $2(16^x) - 5(4^x) + 2 = 0$

(d) $7^{x+1} - 2 = 2(7^x) + 3$

(e) $9^{x+1} + 1 = 10(3^x)$

(f) $6(3^{x-1}) = 3^4 - 3^x$

8. Show that the equation $2^{2x+1} = 3(2^x) + 2$ is satisfied by only one value of x.

9. Solve the following equations.

(a) $8\left(\frac{1}{2}\right)^x + 2 = 2^x$

(b) $4^x = 2^{\frac{3}{x}}$

(c) $3^{x+1}(9^{2-x}) = \frac{1}{3}$

(d) $4^{x+\frac{1}{2}} - 2^{x+3} = 8 + 7(2^x)$

(e) $\frac{5^{x+1} - 20}{10 + 5^x} = 3$

(f) $2^x(5^x) = \frac{1}{100}(10^{x-1})^4$

10. If $\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}$ can be simplified to $\frac{k}{x^3}$, find the values of the constants r and k.

*11. Solve the simultaneous equations $64(4^y) = 16^x$ and $3^y = 4(3^{x-2}) - 1$.

*12. By using an appropriate substitution, find the value of x for which

$$x^{\frac{3}{2}} - 8x^{-\frac{3}{2}} = 7$$

ANSWERS

CHAPTER 1

Exercise 1.1 *(p 2)*

1. (a) 1 (b) 4 (c) 144 (d) $\frac{1}{2}$ (e) 5 (f) $\frac{1}{3}$

2. (a) 54 (b) 4

3. (a) $8y$ (b) $\frac{1}{2}y^2$ (c) $\frac{12}{y}$ (d) $\frac{32}{y^5}$ (e) $\frac{1}{2}y^2$ (f) $y^3 - \frac{1}{y^2}$

4. (a) $18y^2$ (b) $\frac{1}{6}yz$ (c) $\frac{z^2}{y}$

Exercise 1.2 *(p 5)*

1. (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) -2 (d) $-\frac{5}{2}$ (e) $\frac{1}{2}$ (f) 2
   (g) 2, -2 (h) 2 (i) $\frac{2}{3}$ (j) $\frac{5}{4}$ (k) 2 (l) -6, 2
   (m) -2, 4 (n) -2, 1
2. $a = 3$, $n = 2$
3. (a) $x = -\frac{4}{9}$, $y = \frac{1}{9}$ (b) $x = 2$, $y = 1$ (c) $x = 1$, $y = -1$
4. $m = 4$, $n = 3$ **5.** 3
6. (a) 0, 1 (b) 0, 2
7. (a) 0, 1 (b) 1, 3 (c) $-\frac{1}{2}$, $\frac{1}{2}$ (d) 0 (e) -2, 0 (f) 3
9. (a) 2 (b) ± 2 (c) 6 (d) 3 (e) 2 (f) 2
10. $r = 3$, $k = \frac{2}{3}$ **11.** $x = 1$, $y = -1$ or $x = 2$, $y = 1$ **12.** 4

Exercise 1.3 *(p 9)*

1. (a) $4 = \log_2 16$ (b) $-2 = \log_3\left(\frac{1}{9}\right)$ (c) $2 = \log_{10} 100$
   (d) $3 = \log_a y$ (e) $x = \log_2 p$ (f) $4 = \log_a(2 - k)$
2. (a) $5^3 = 125$ (b) $2^{-2} = \frac{1}{4}$ (c) $4^3 = 64$
   (d) $x^4 = 3$ (e) $3^n = y$ (f) $2^{p+1} = 4y$
3. (c), (d) and (e) are not defined
4. (a) 8 (b) 3 (c) $\frac{3}{2}$ (d) 5
   (e) $-\frac{7}{16}$ (f) $\frac{3}{4}$ (g) 2, 4 (h) 4
5. (a) -2 (b) 2 (c) 8 (d) $\frac{25}{16}$ (e) 0
6. 2 **7.** 3^{a+b}
8. (a) $x = 2$, $y = 4$ (b) $x = 4$, $y = 2$ **10.** 9